

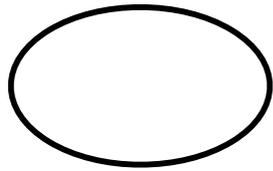
# Introduction to Petri Nets

Tiankuang Zhang (Ty) z5236826

# Welcome Guys :D

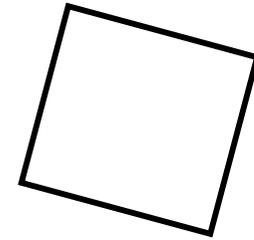
- Part 1: Intuitive Understanding of Petri nets
- Part 2: Basic Concepts
- Part 3: Elementary system nets
- Part 4: Sth interesting if we have time

# Part 1: Intuitive Understanding



Place

Transition



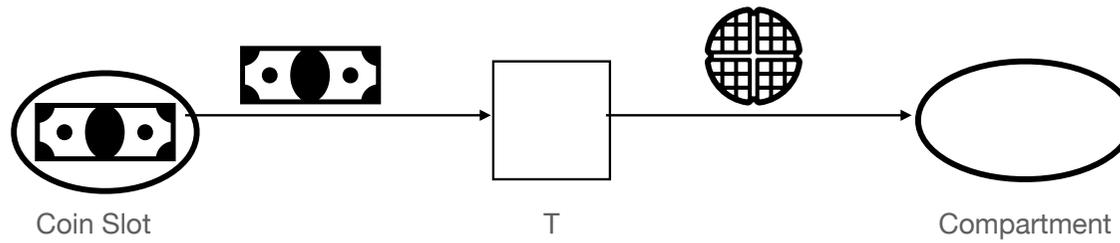
## 4 Components

Arc

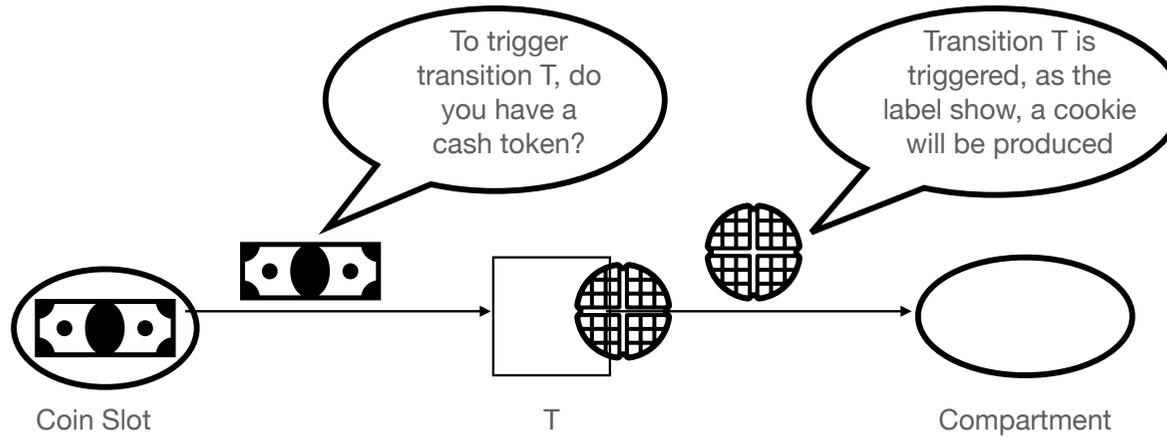


Token

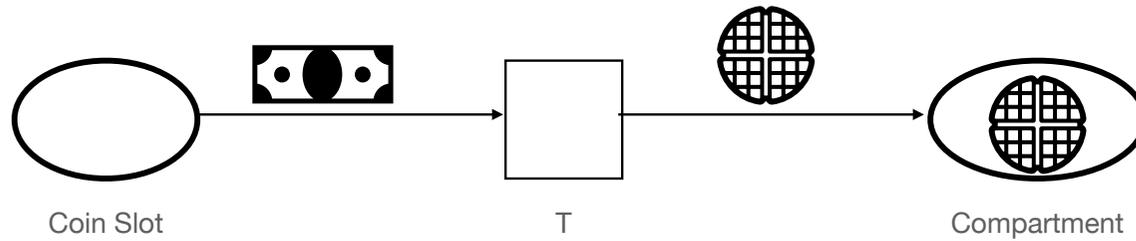
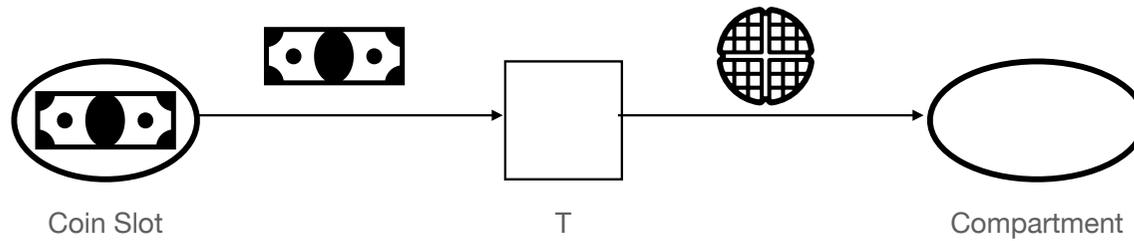
# Part 1: Intuitive Understanding



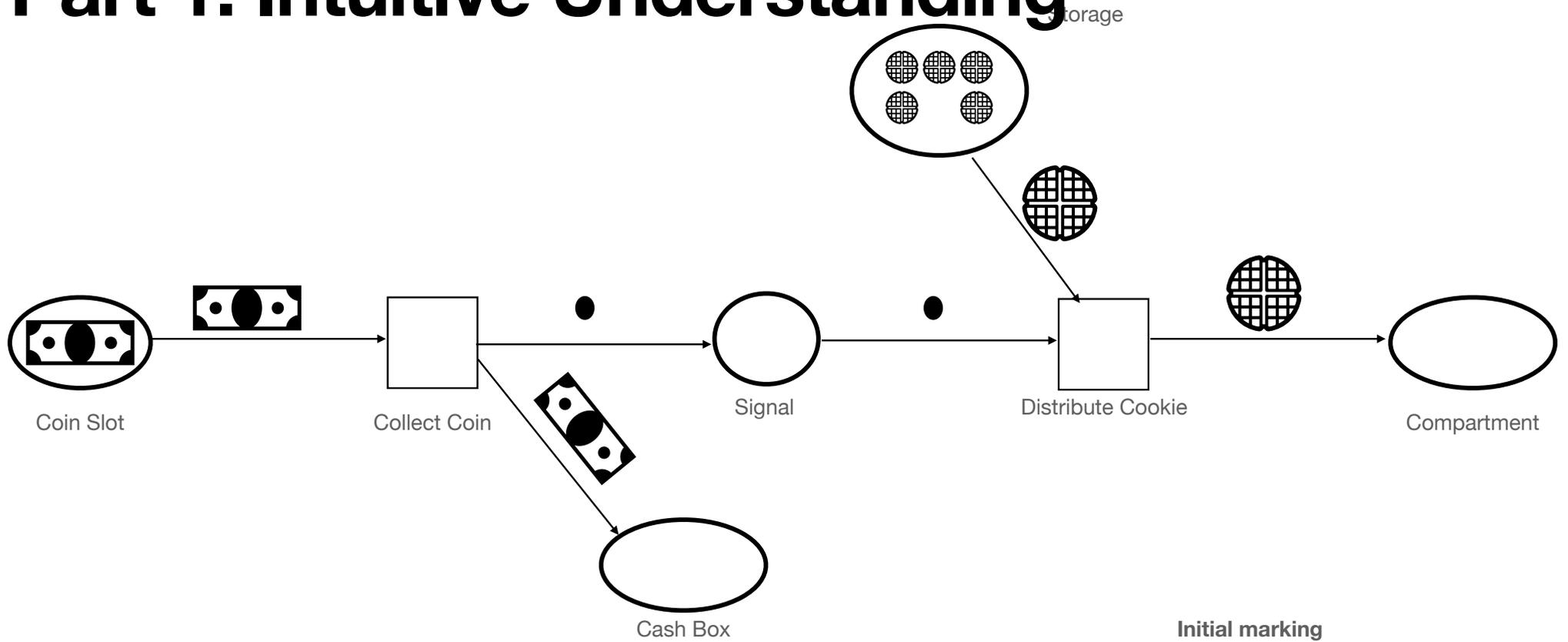
# Part 1: Intuitive Understanding



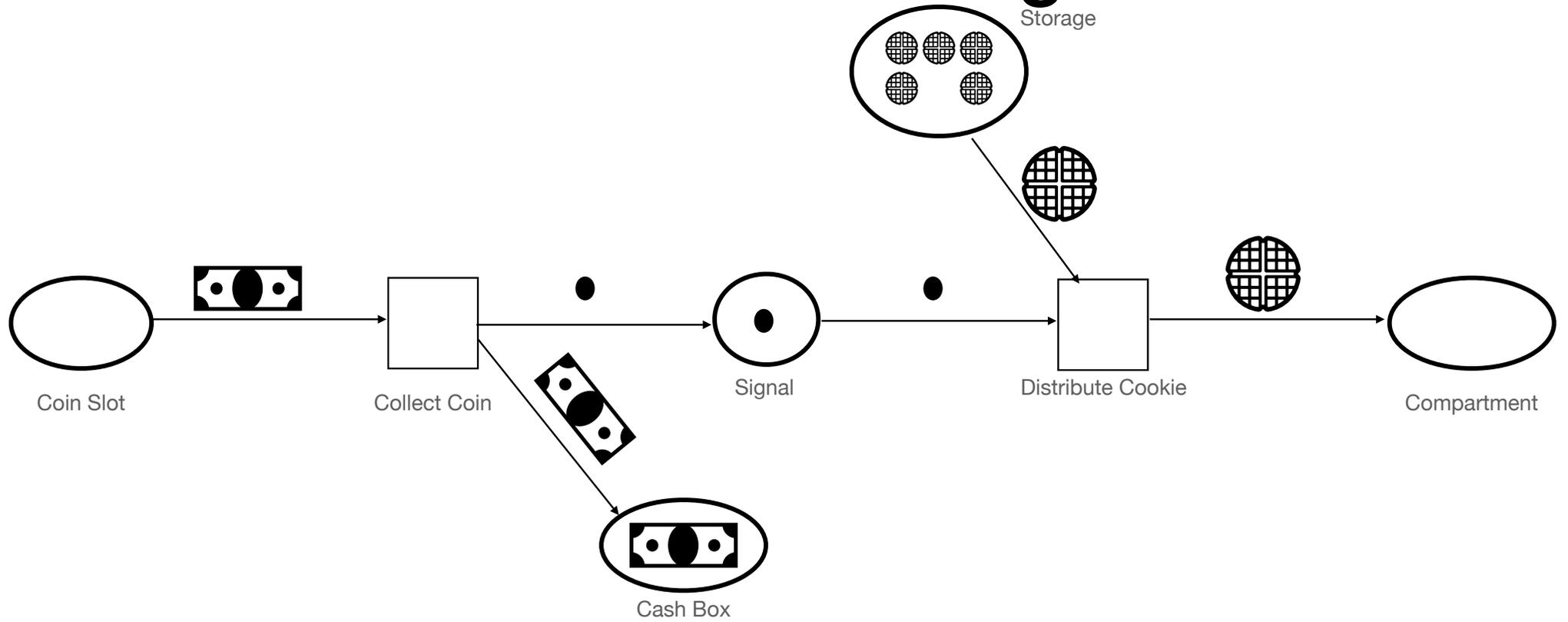
# Part 1: Intuitive Understanding Marking = State



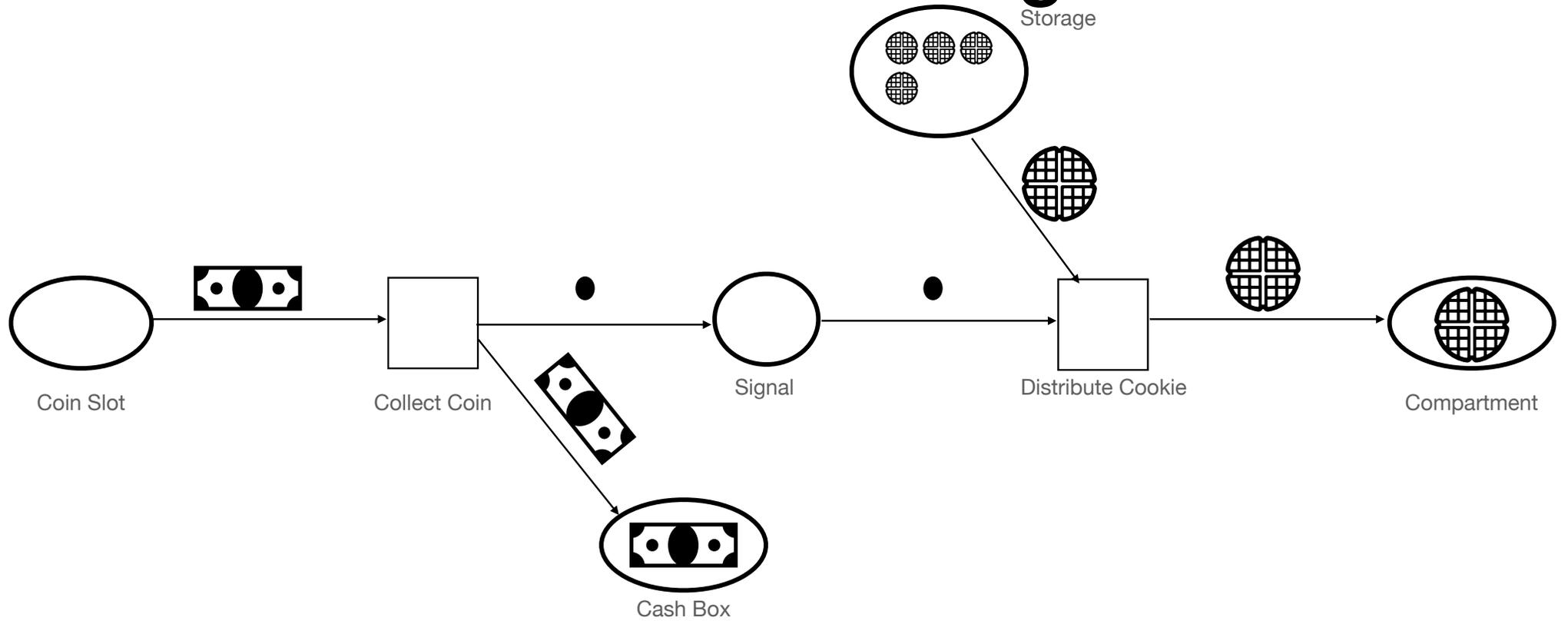
# Part 1: Intuitive Understanding



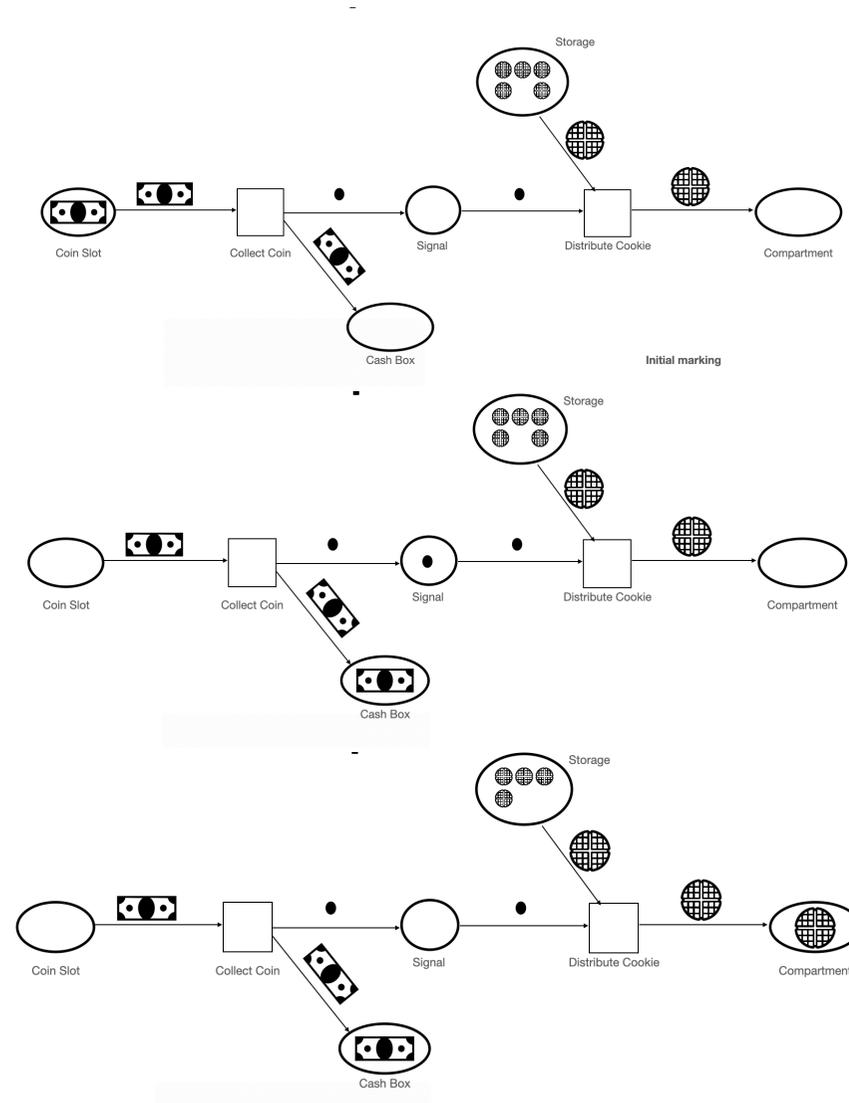
# Part 1: Intuitive Understanding



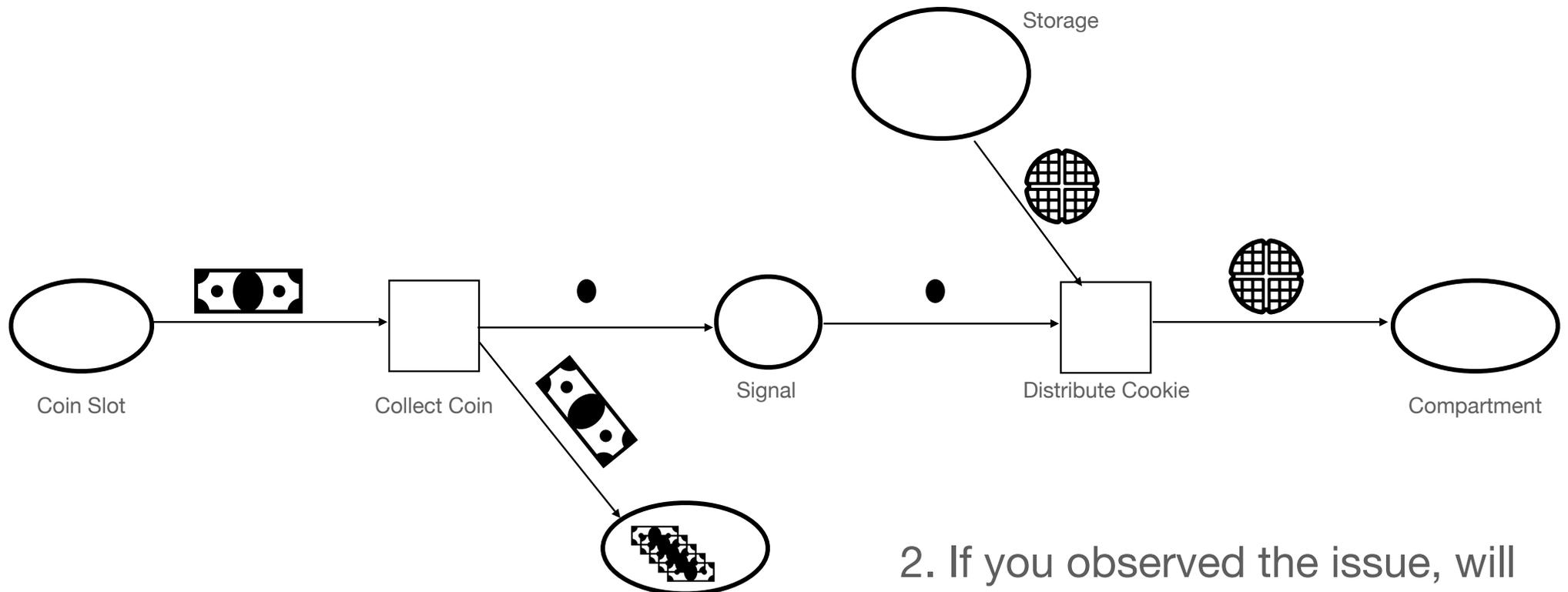
# Part 1: Intuitive Understanding



# Part 1: Intuitive Understanding



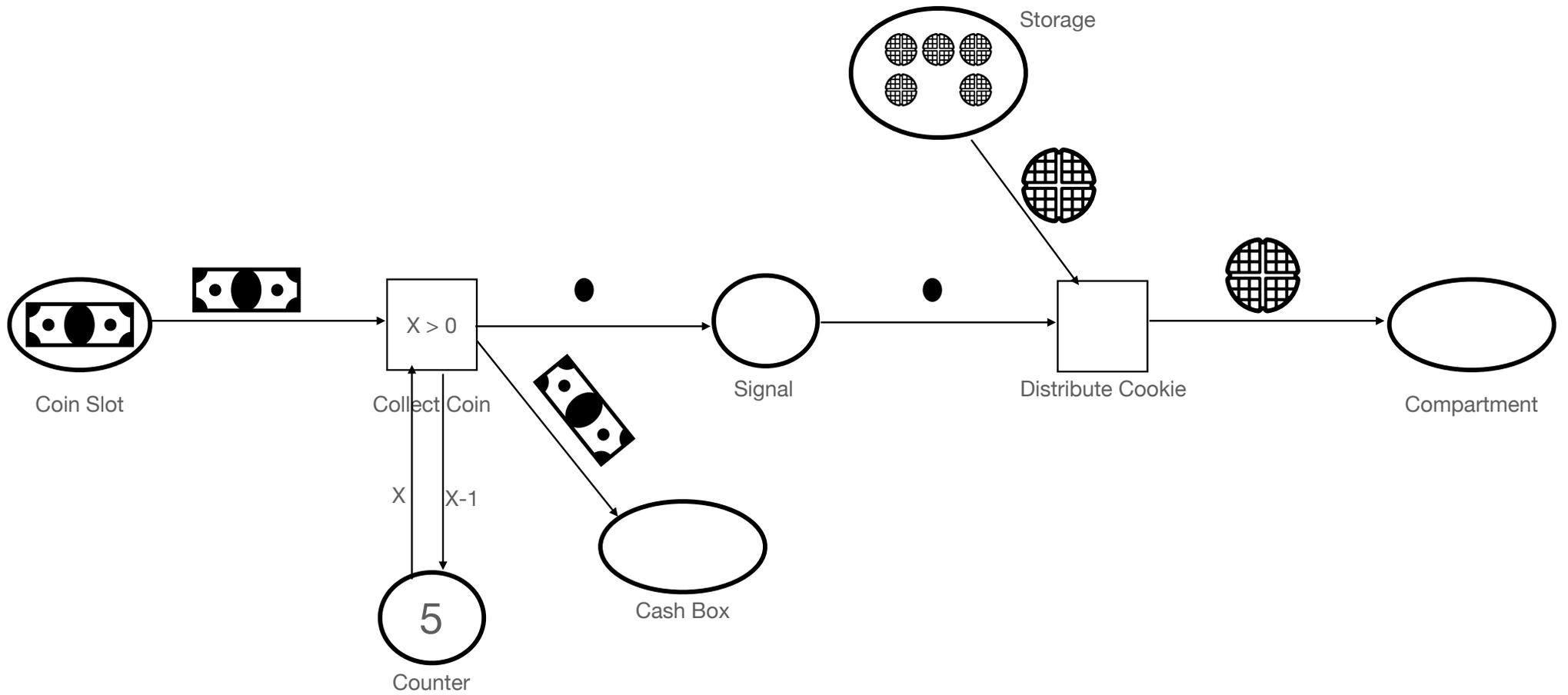
# QUESTION TIME



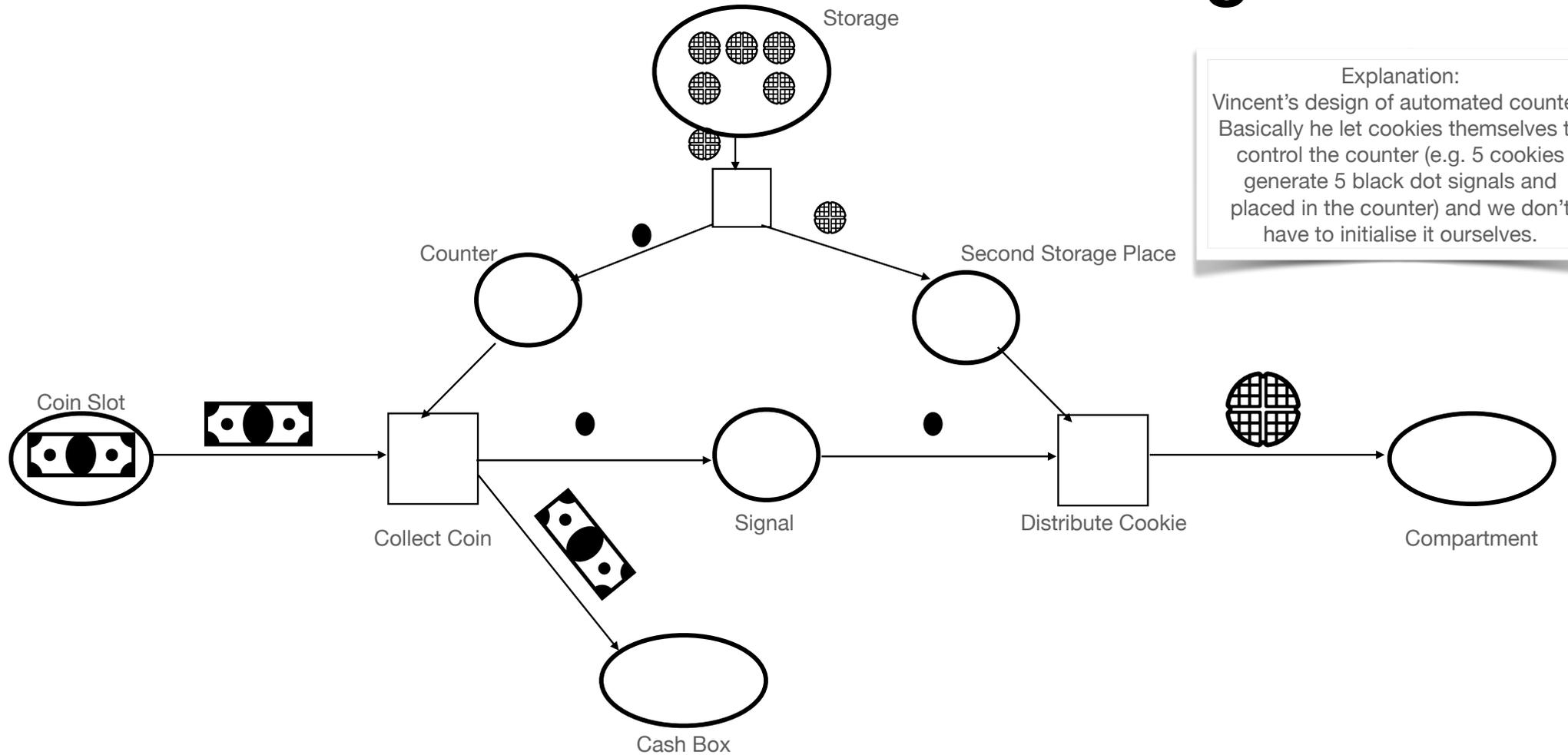
1. Now you've inserted 5 coins, will you insert a 6th coin?

2. If you observed the issue, will you stop transition collect coin or the transition distribute cookie?

# QUESTION TIME

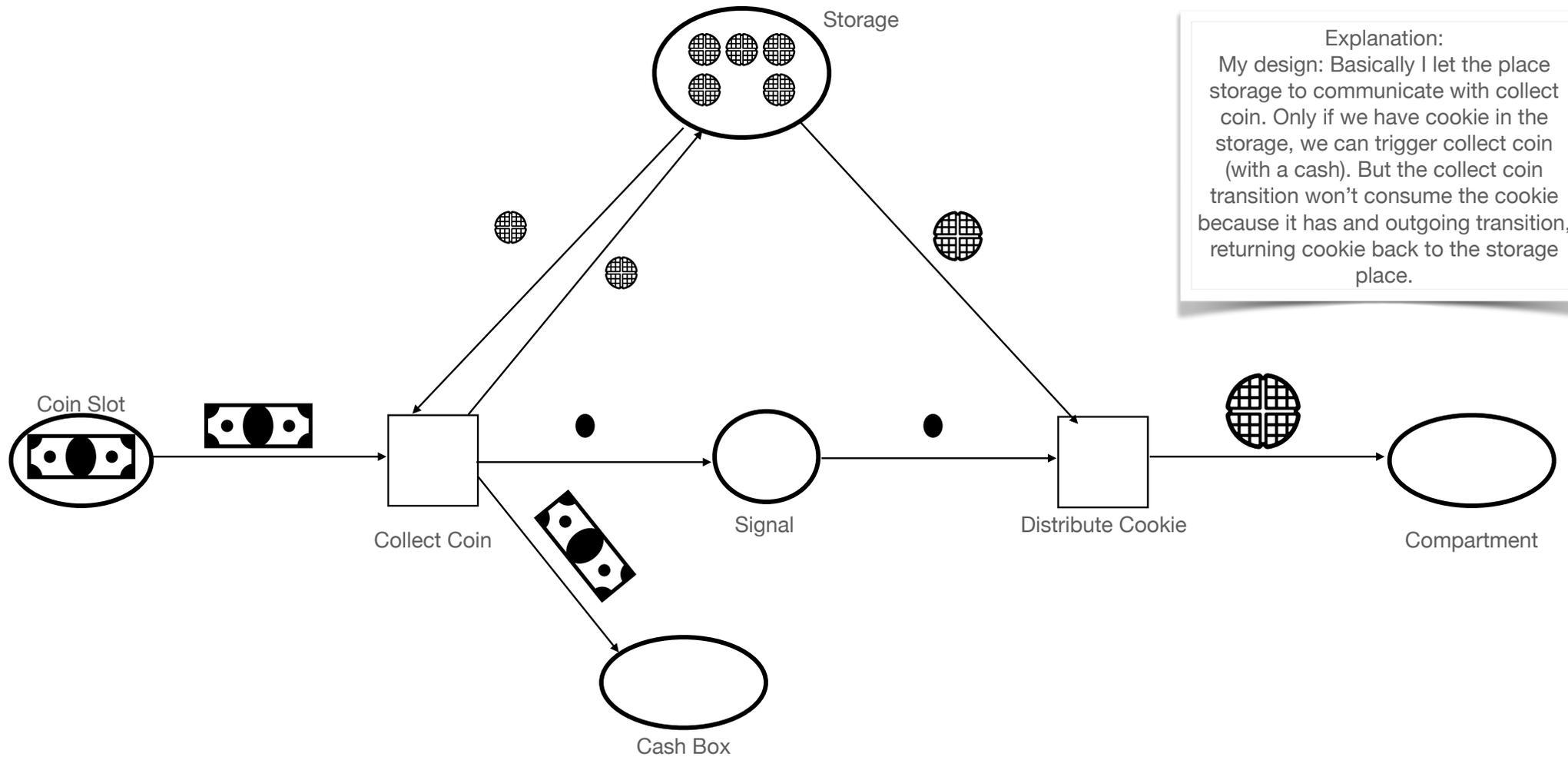


# Vincent's Automated Counter Design



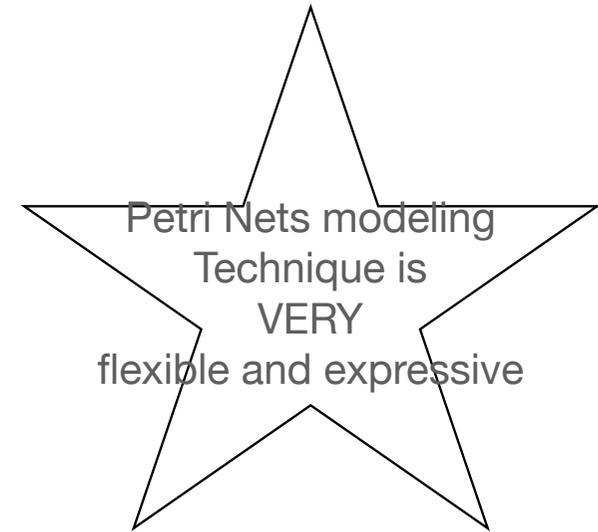
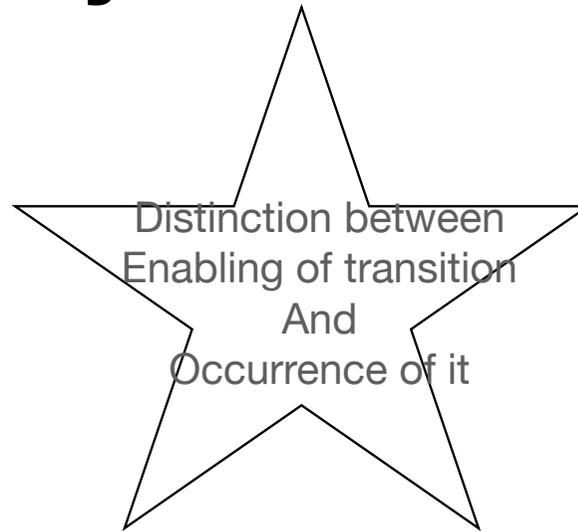
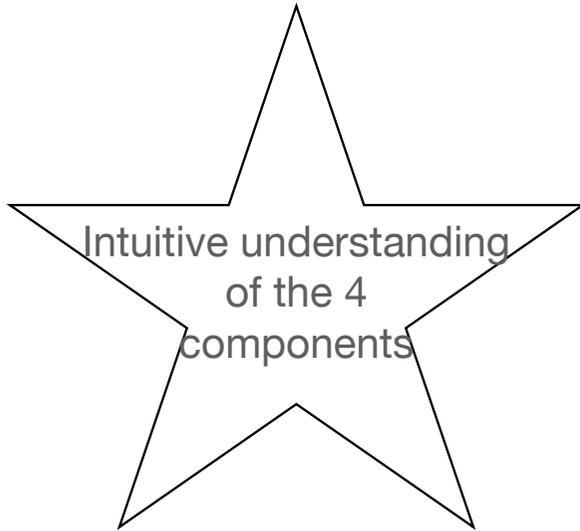
Explanation:  
Vincent's design of automated counter:  
Basically he let cookies themselves to control the counter (e.g. 5 cookies generate 5 black dot signals and placed in the counter) and we don't have to initialise it ourselves.

# My Alternative design to this issue (without counter)

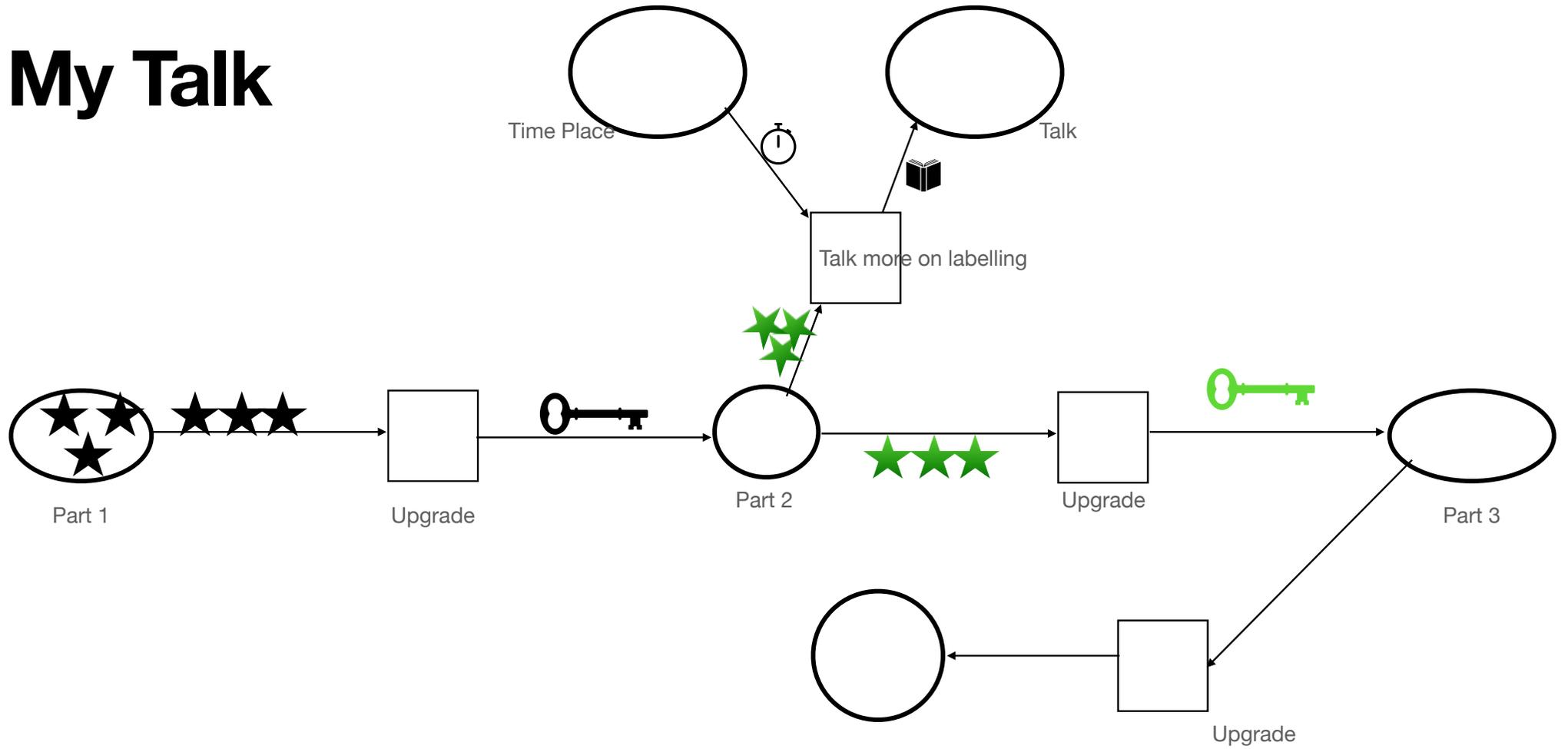


Explanation:  
My design: Basically I let the place storage to communicate with collect coin. Only if we have cookie in the storage, we can trigger collect coin (with a cash). But the collect coin transition won't consume the cookie because it has an outgoing transition, returning cookie back to the storage place.

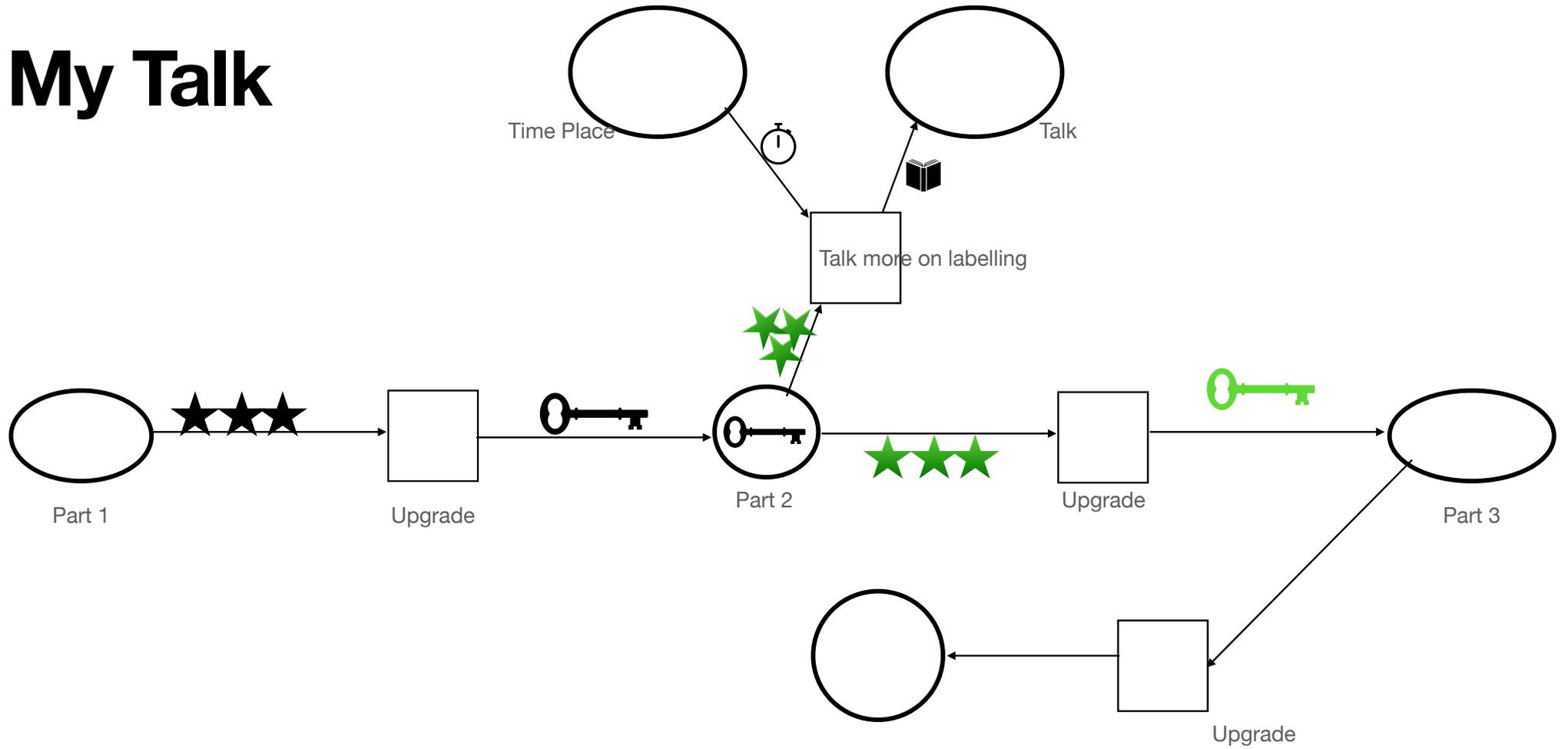
# Part 1 Take Away



# My Talk



# My Talk



# Part 2: Basic Concepts

How is a place different from a transition?

What is a place?

Can an arc point from a transition to another transition?

Can an arc point from a place to another place?

How do you know if you model sth correctly?

Do you always have to draw a lot to model sth?

How is petri nets different from other modelling technique?

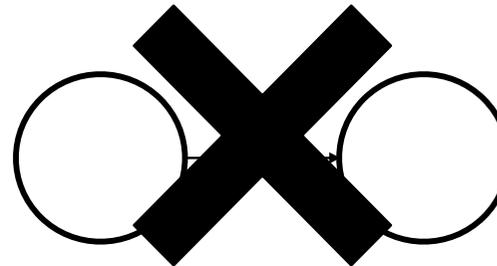
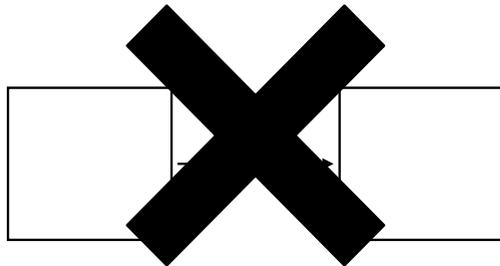
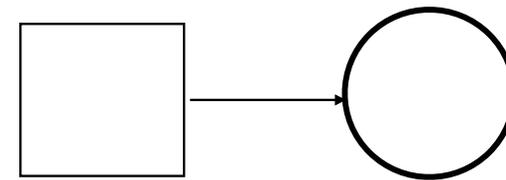
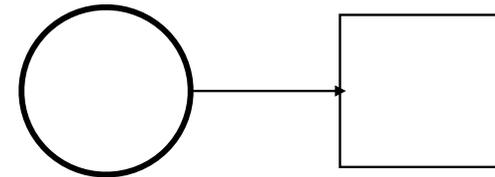
# Part 2: Basic Concepts

- **Place:** passive component
  - Store
  - Accumulate
  - Show
- **Transition:** active component
  - Consume
  - Transport
  - Produce
  - Change

**Not A STATE**

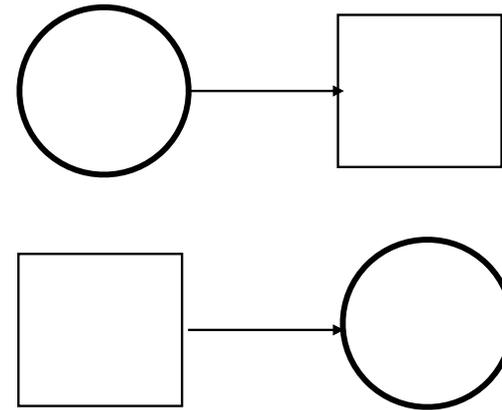
# Part 2: Basic Concepts

- Arc
- Bipartite Graph
- “Either an arc runs from a place to a transition or the other way around” (Wolfgang, 14)

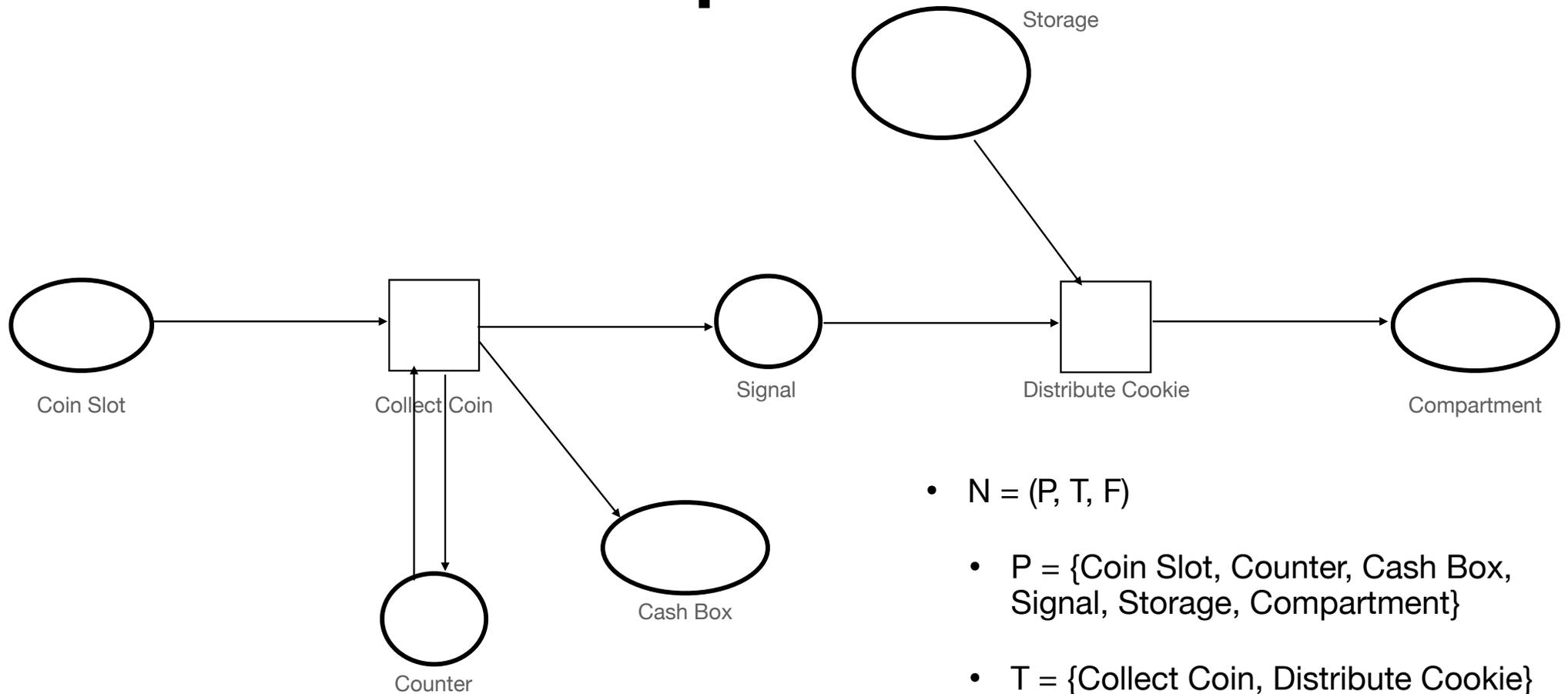


# Part 2: Basic Concepts

- Net Structure
  - $N = (P, T, F)$ 
    - P: set of all places
    - T: set of all transitions
    - $F \subseteq (P \times T) \cup (T \times P)$



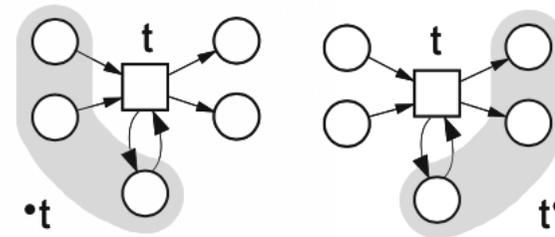
# Part 2: Basic Concepts



- $N = (P, T, F)$
- $P = \{\text{Coin Slot, Counter, Cash Box, Signal, Storage, Compartment}\}$
- $T = \{\text{Collect Coin, Distribute Cookie}\}$
- $F \subseteq (P \times T) \cup (T \times P)$

# Part 2: Basic Concepts

- Pre-set and Post-set
  - In an unambiguously defined Net structure  $N$ , for a component  $x$  (place/transition) we can define
    - Pre-set of  $x$ :  $\bullet x =_{\text{def}} \{y \mid yF x\}$
    - Post-set of  $x$ :  $x^\bullet =_{\text{def}} \{y \mid xF y\}$ .
    - Loop:  $x \in \bullet y$  and  $y \in \bullet x$

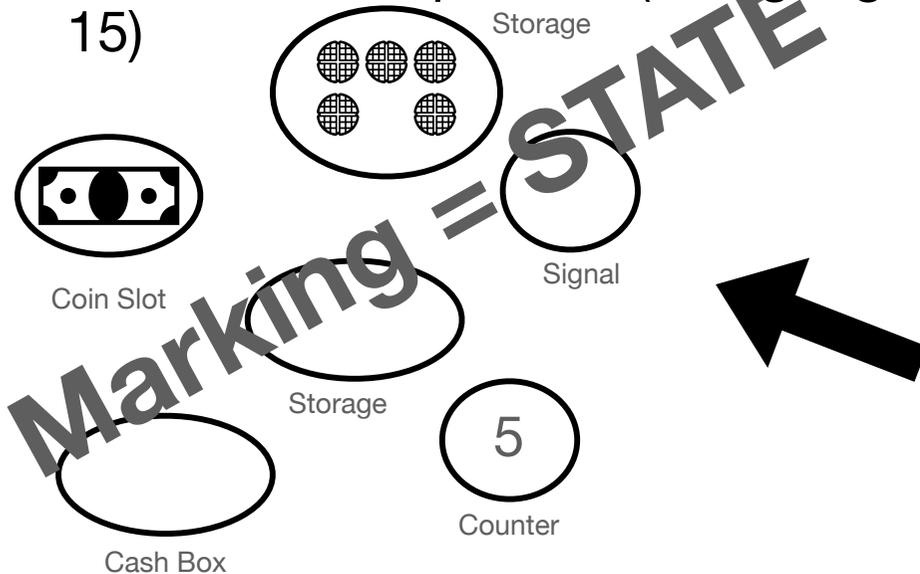


(Wolfgang, 15)

# Part 2: Basic Concepts

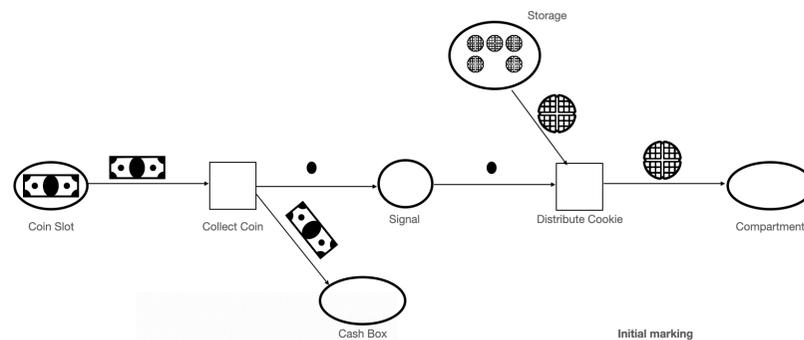
- Marking

- “A marking is a distribution of tokens across places.” (Wolfgang, 15)



- Note:

- All places must be considered.
- Marking can be represented graphically. But not necessarily.



# Part 2: Basic Concepts

- Multiset
  - Example: mixed kinds of tokens in a place.
    - [🌐🌐🌐🌐🌐]
  - A multiset  $a$  is formally a mapping
  - $a : U \rightarrow N$
- Domain: Universe
  - Sufficiently Large
  - Collection of all examined tokens
  - E.g {🌐 🇺🇸 ● 0 1 2 3 4 5 }
- Codomain: Natural Number

# Part 2: Basic Concepts

- Multiset Example:

- $U = \{ \text{⊗} \text{⊠} \bullet 0 1 2 3 4 5 \}$

- $R = [ \text{⊗} \text{⊗} \text{⊗} \text{⊠} \text{⊠} \bullet ]$

- $a(\text{⊠}) = 2$

- $a(\text{⊗}) = 3$

- $a(\bullet) = 1$

- $a(u) = 0$  for any other  $u$  in  $U$

- $a(3) = 0$

- $a(2) = 0$

# Part 2: Basic Concepts

- $\mathcal{M}(U)$ : a set of all multisets over  $U$
- When  $U$  can be unambiguously identified, we write  $\mathcal{M}(U)$
- Otherwise, we just write  $\mathcal{M}$ .

What about elementary system nets?  
- What is  $U$ ?  
- What is  $\mathcal{M}(U)$ ?  
- Assume we allow at most one black dot in one place,  
- do we still need the data structure multiset?

- Example:  $U = \{\text{[grid]}, \text{[black dot]}, \text{[white dot]}\}$
- $\mathcal{M}(U) = \{[], \text{[grid]}, \text{[black dot]}, \text{[white dot]}, \text{[grid, grid]}, \text{[grid, black dot]}, \text{[grid, white dot]}, \text{[black dot, black dot]}, \text{[black dot, white dot]}, \text{[white dot, white dot]}, \dots\}$
- A multiset  $a$  is finite if
  - $a(u) \neq 0$  for only finite number of  $u \in U$ .
  - $+$ ,  $-$ ,  $\geq$ ,  $\leq$ ,  $=$

# Part 2: Basic Concepts

- Marking: with the help of multisets:

$M: P \rightarrow \mathcal{M}(U)$

- $M_0(\text{coin slot}) = [\text{coin}]$

- $a(\text{coin}) = 1$

- $M_0(\text{cash box}) = M_0(\text{signal}) = M_0(\text{compartment})$

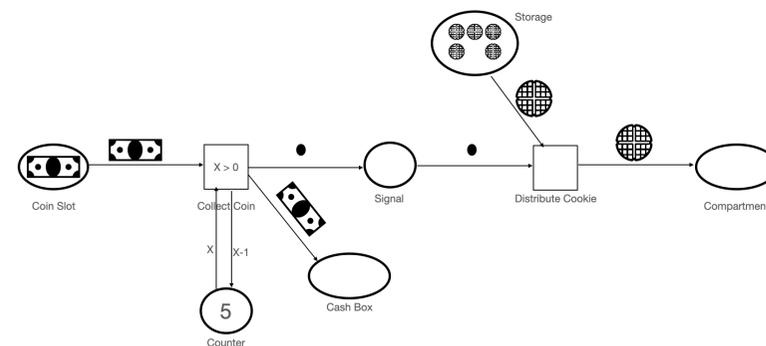
- $a() = 0$

- $M_0(\text{storage}) = [5 \text{ cookies}]$

- $a(\text{cookie}) = 5$

- $M_0(\text{counter}) = [5]$

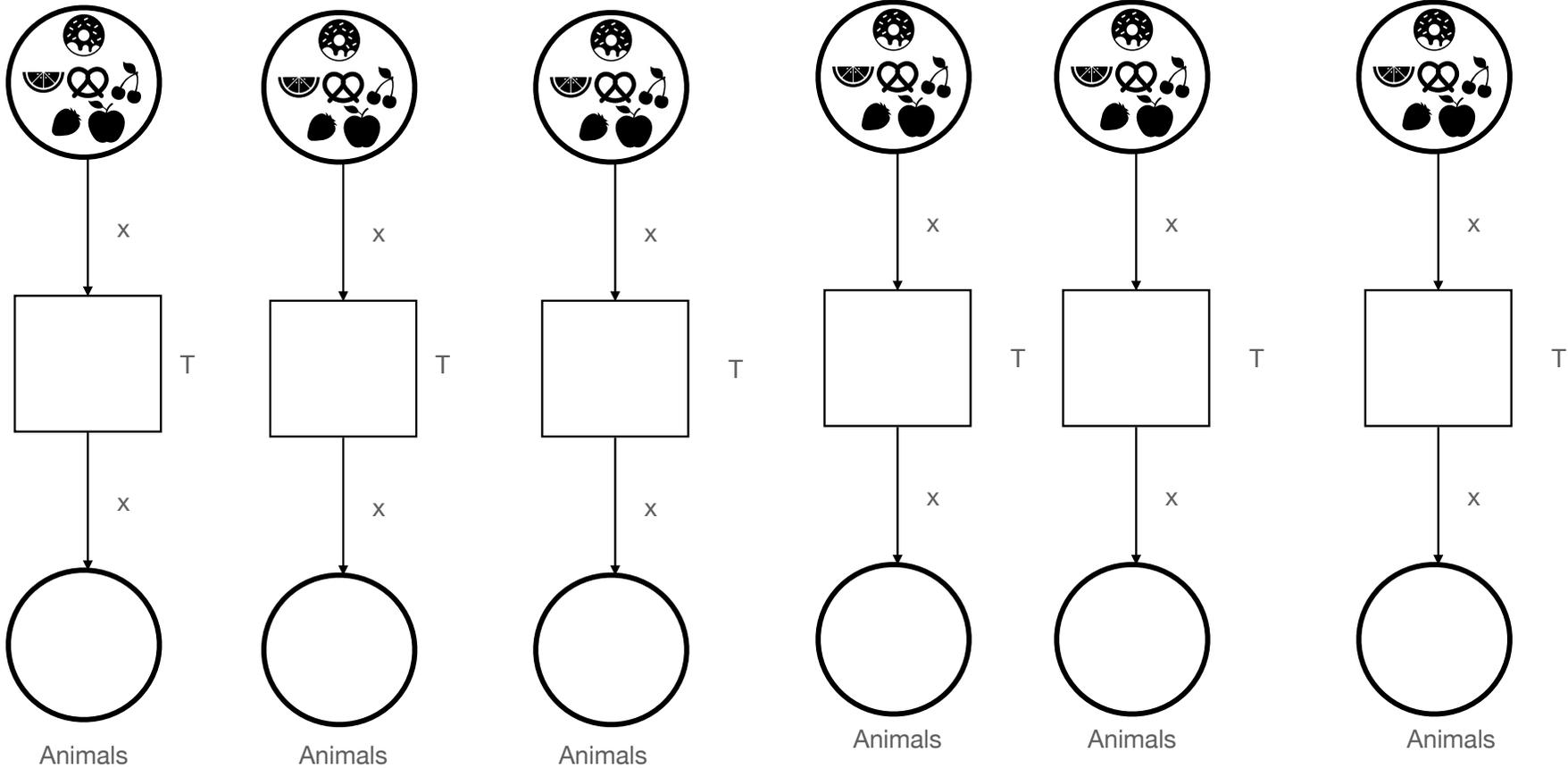
- $a(5) = 1$



# Part 2: Basic Concepts

- System Net
  - Net structure + Initial Marking + Transition condition (labelling in transition) + Arc\_labellings + cold\_transition
- Reachability of Marking:  
reachability is not so different from reachability of states in process graph especially in part 3.
- Final Marking

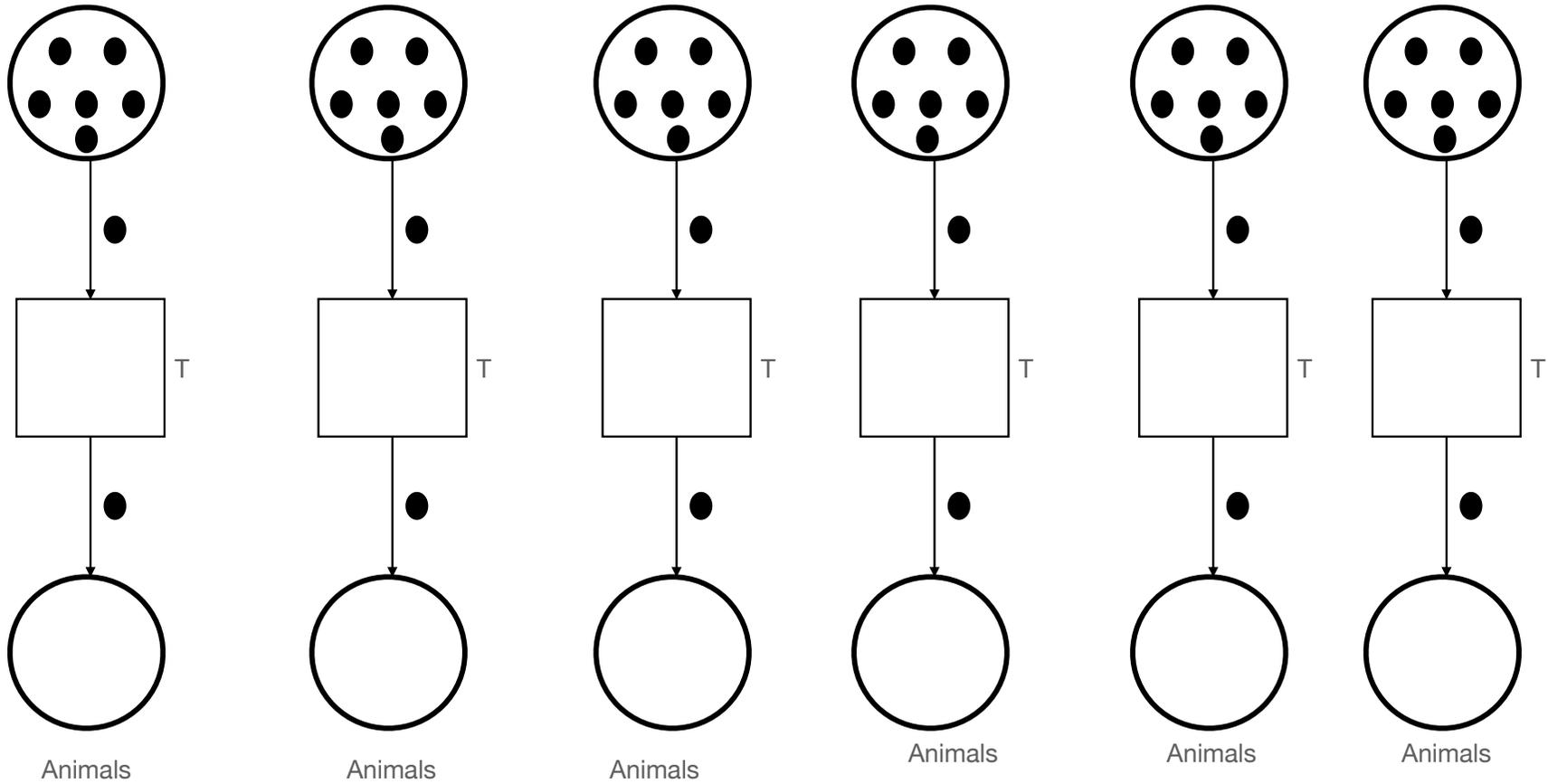
# QUESTION TIME



How many markings are there from the initial marking?

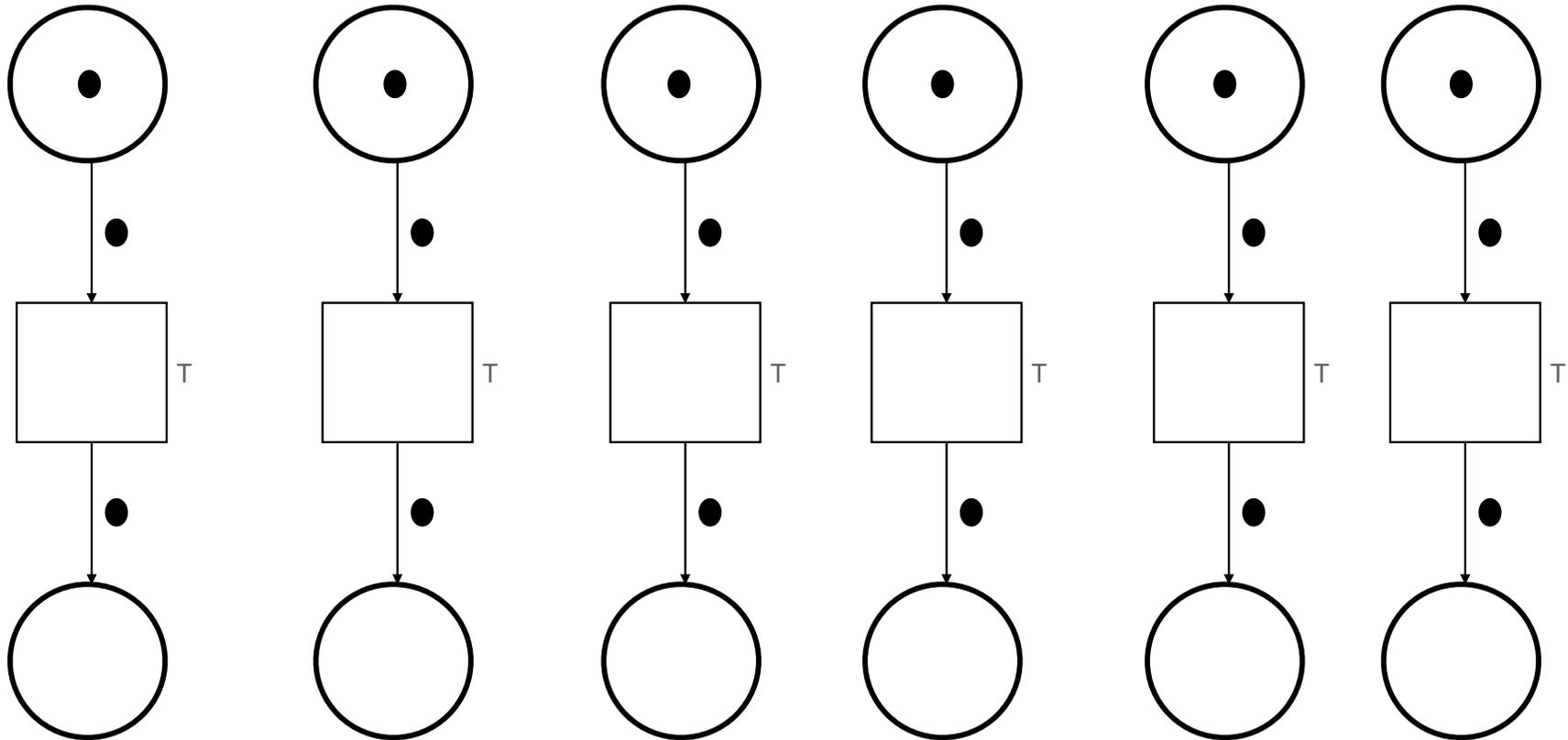
# QUESTION TIME

What about now?

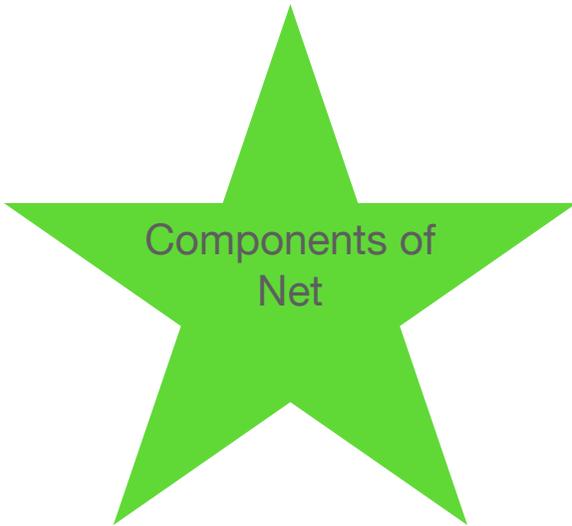


# QUESTION TIME

What about now?



# Part 2 Take Away



Components of  
Net

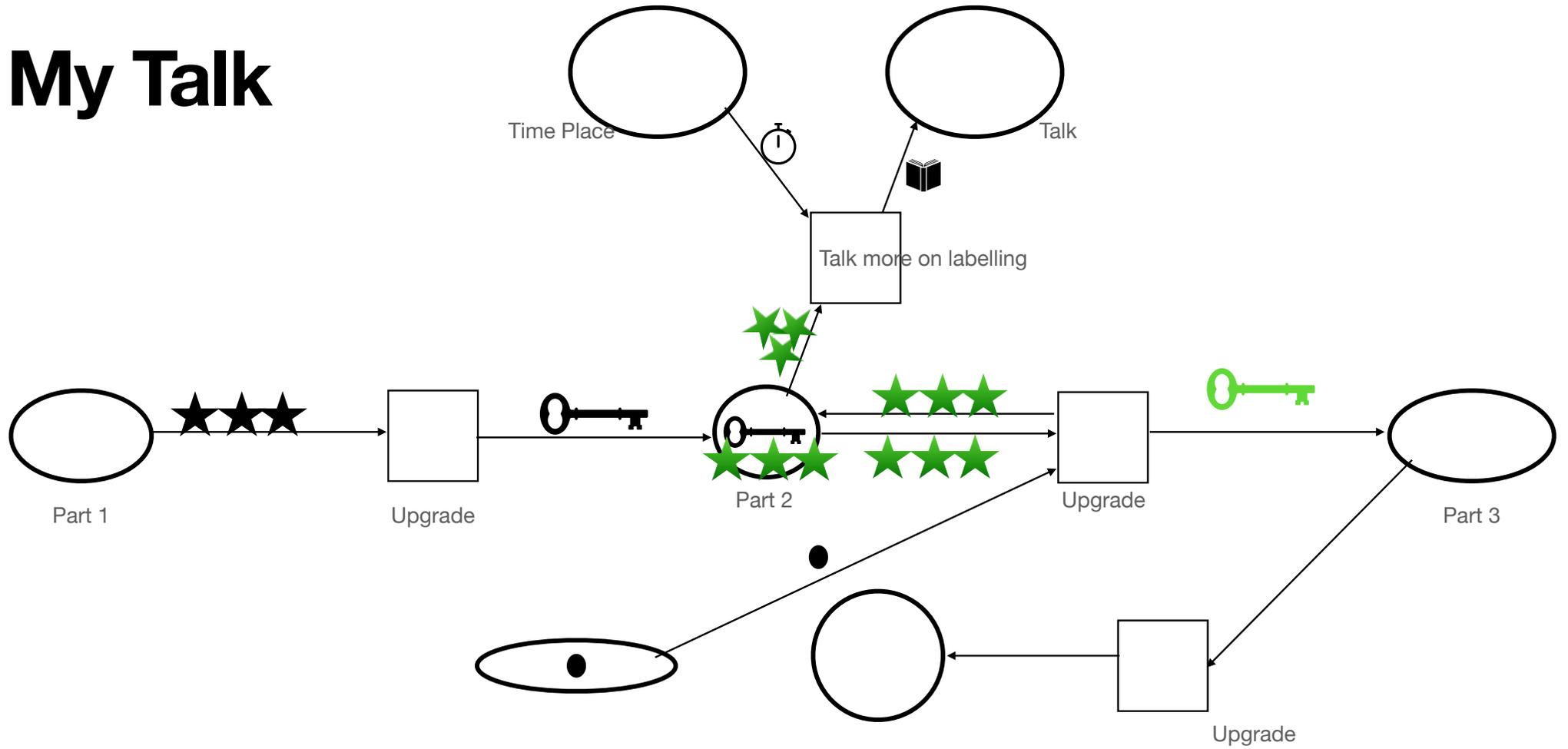


Multiset as special data  
Structure used  
By Petri Nets

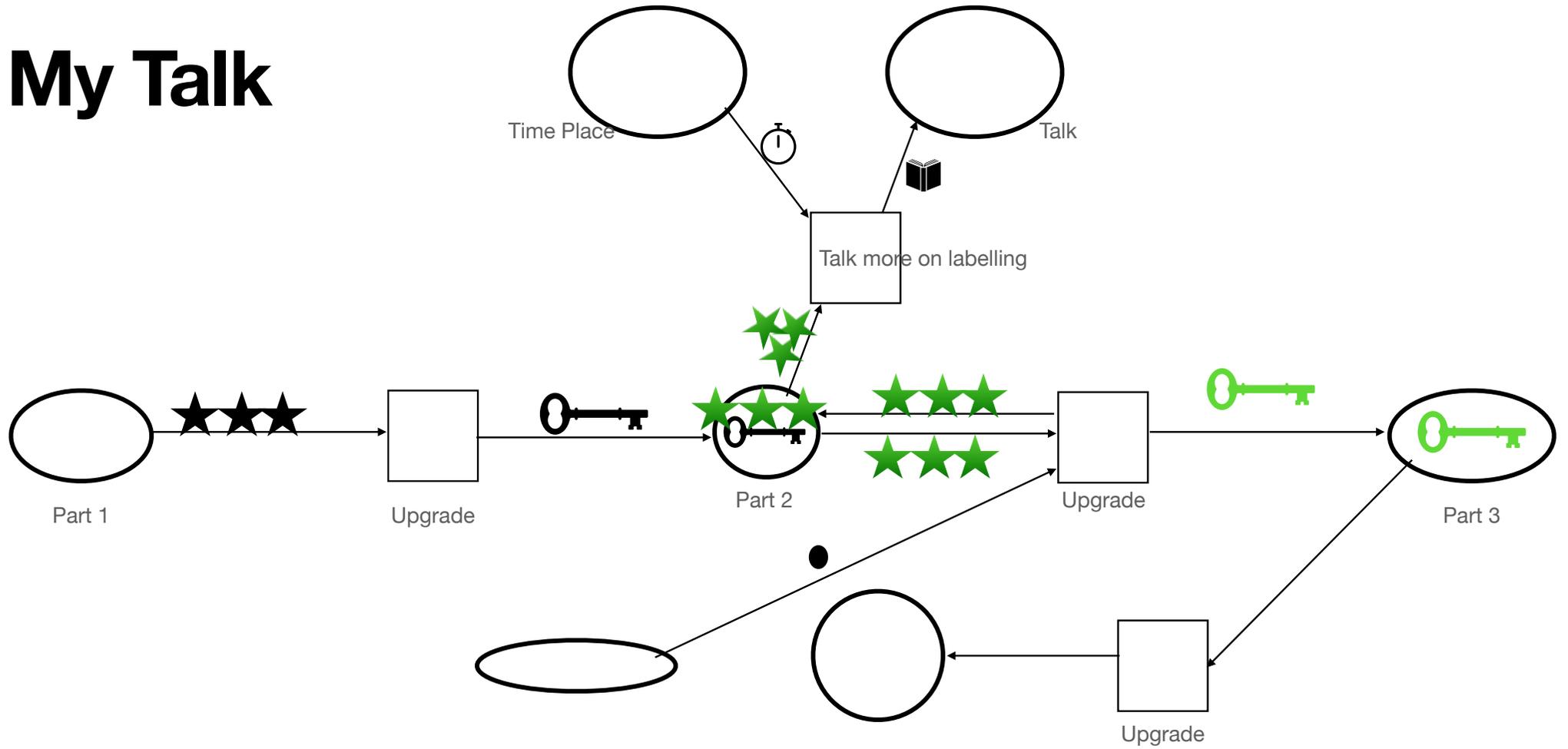


Non-graphical  
representation  
Of Marking

# My Talk



# My Talk

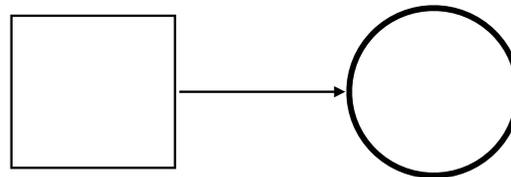
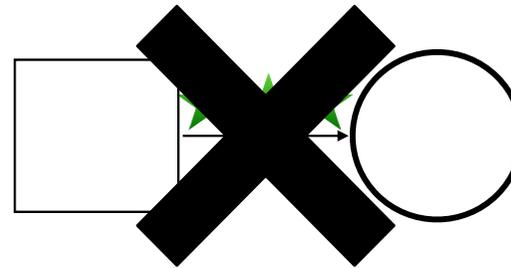


# Part 3: Elementary system nets

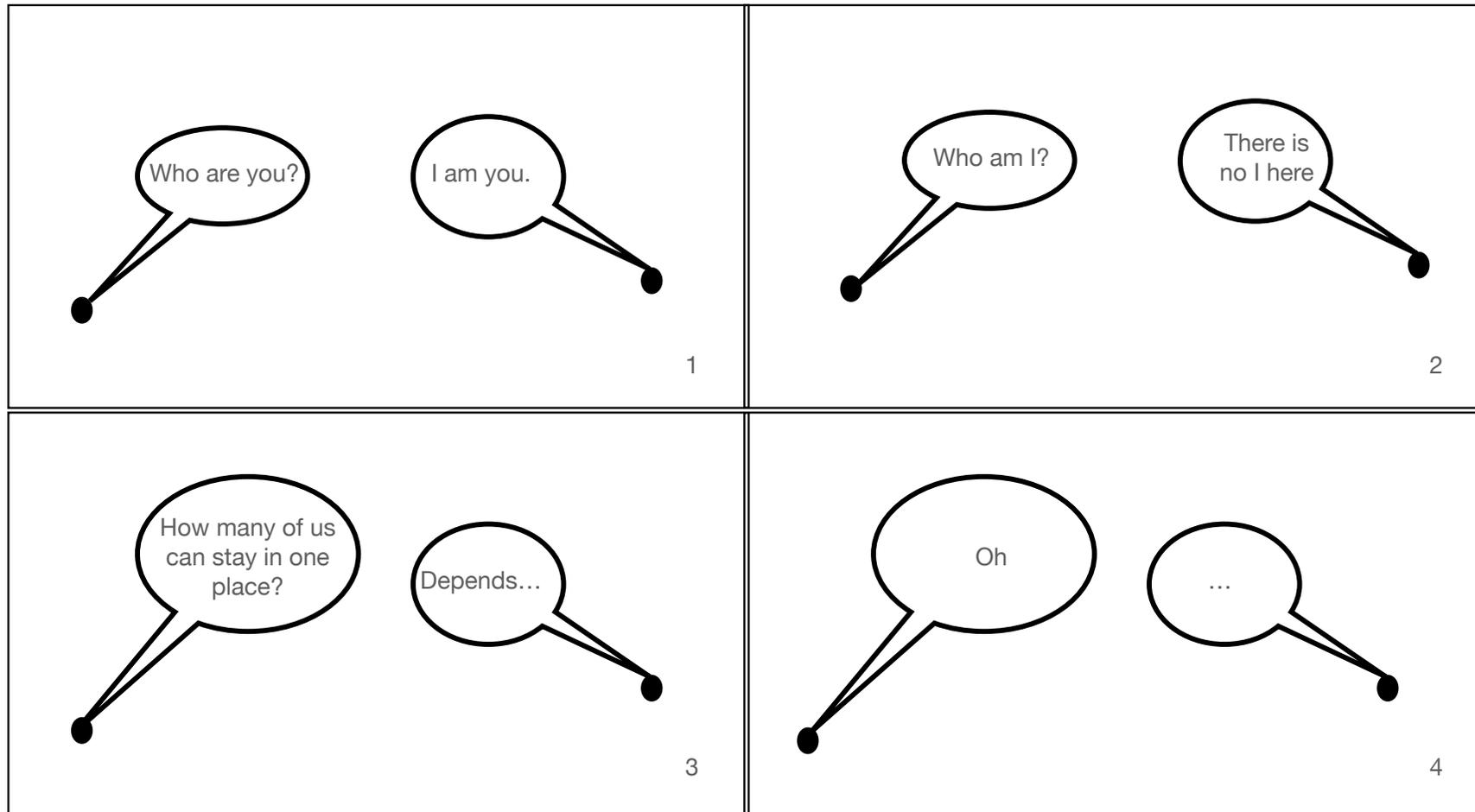
- Part1, Part2: Generic System Nets
- Part3: Elementary System Nets

# Part 3: Elementary system nets

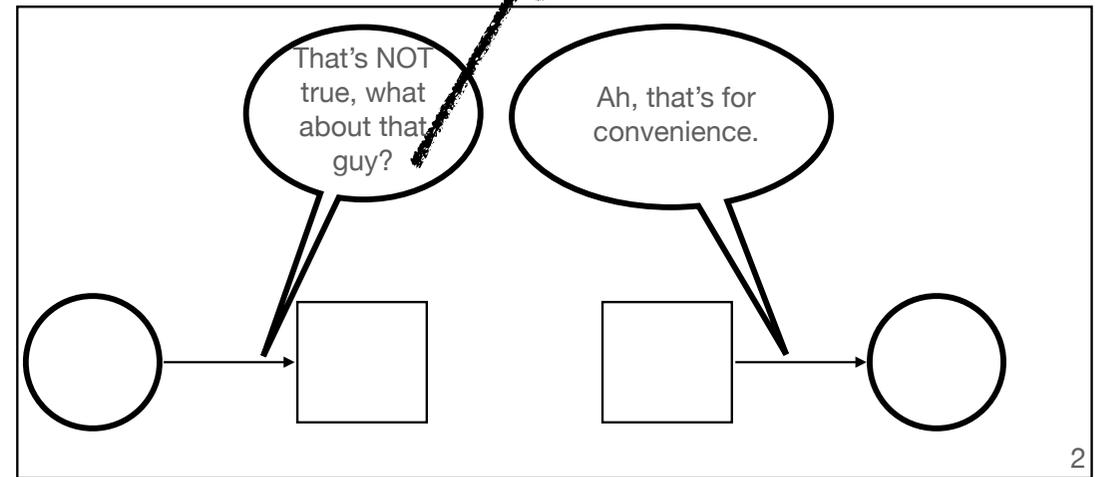
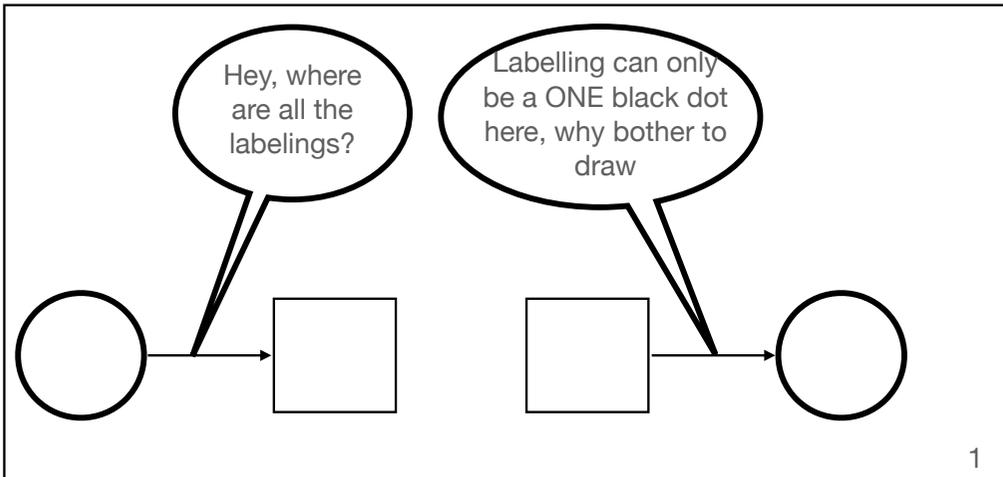
- 2 Difference
  - Abstract Black Dot Token ONLY
  - No labelling: default is ●



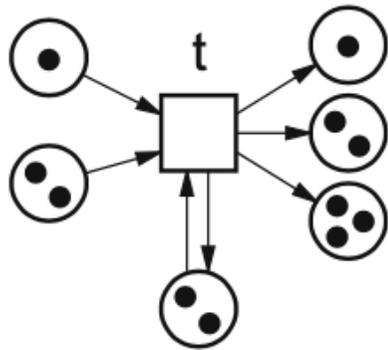
# Part 3: Elementary system nets



# Part 3: Elementary system nets



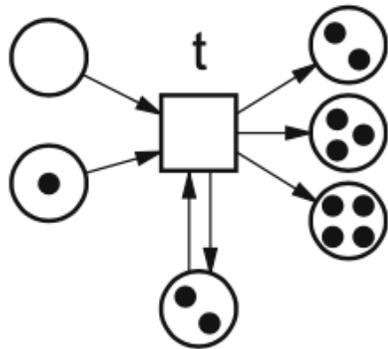
# Part 3: Elementary system nets



$$M'(p) = \begin{cases} M(p) - 1 & \text{if } p \in \bullet t \text{ and } p \notin t^\bullet \\ M(p) + 1 & \text{if } p \in t^\bullet \text{ and } p \notin \bullet t \\ M(p) & \text{otherwise} \end{cases}$$

$t$   
→

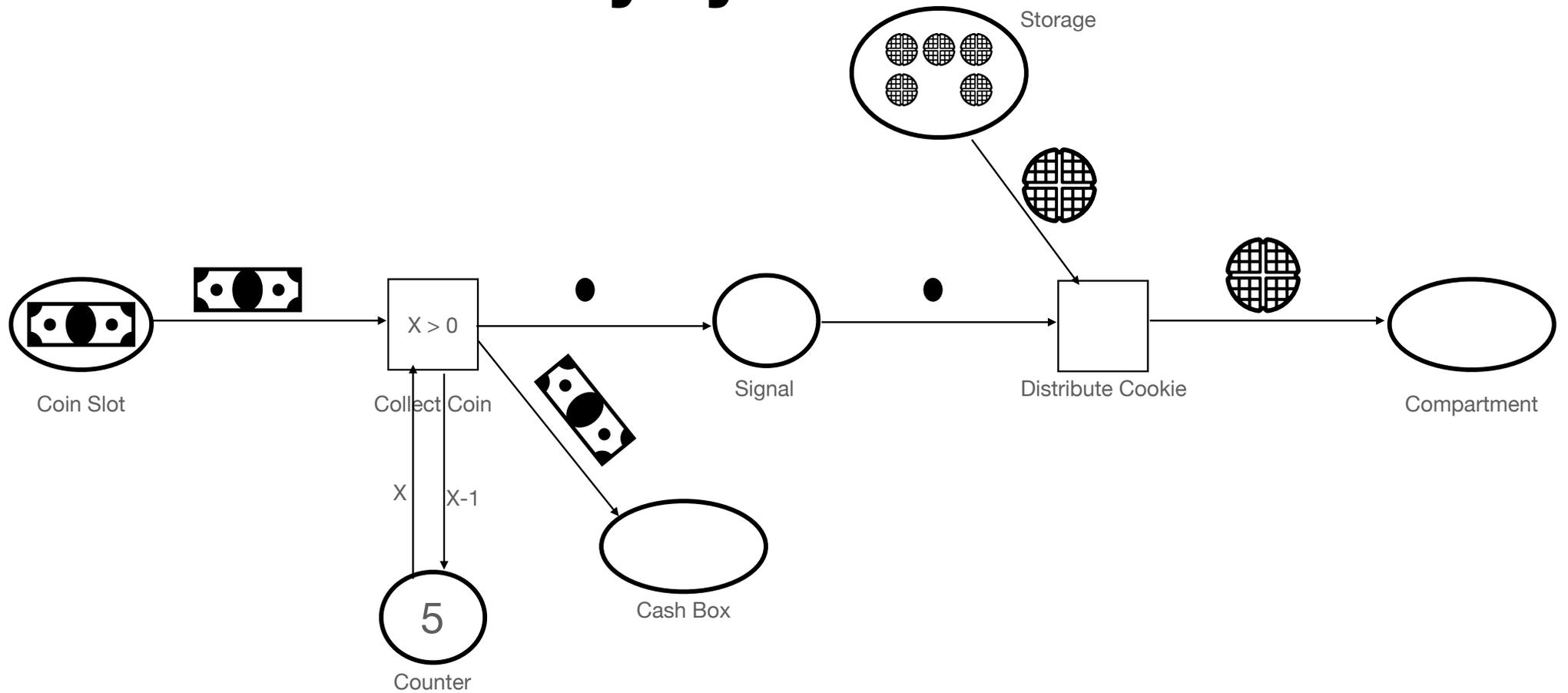
Step Rule



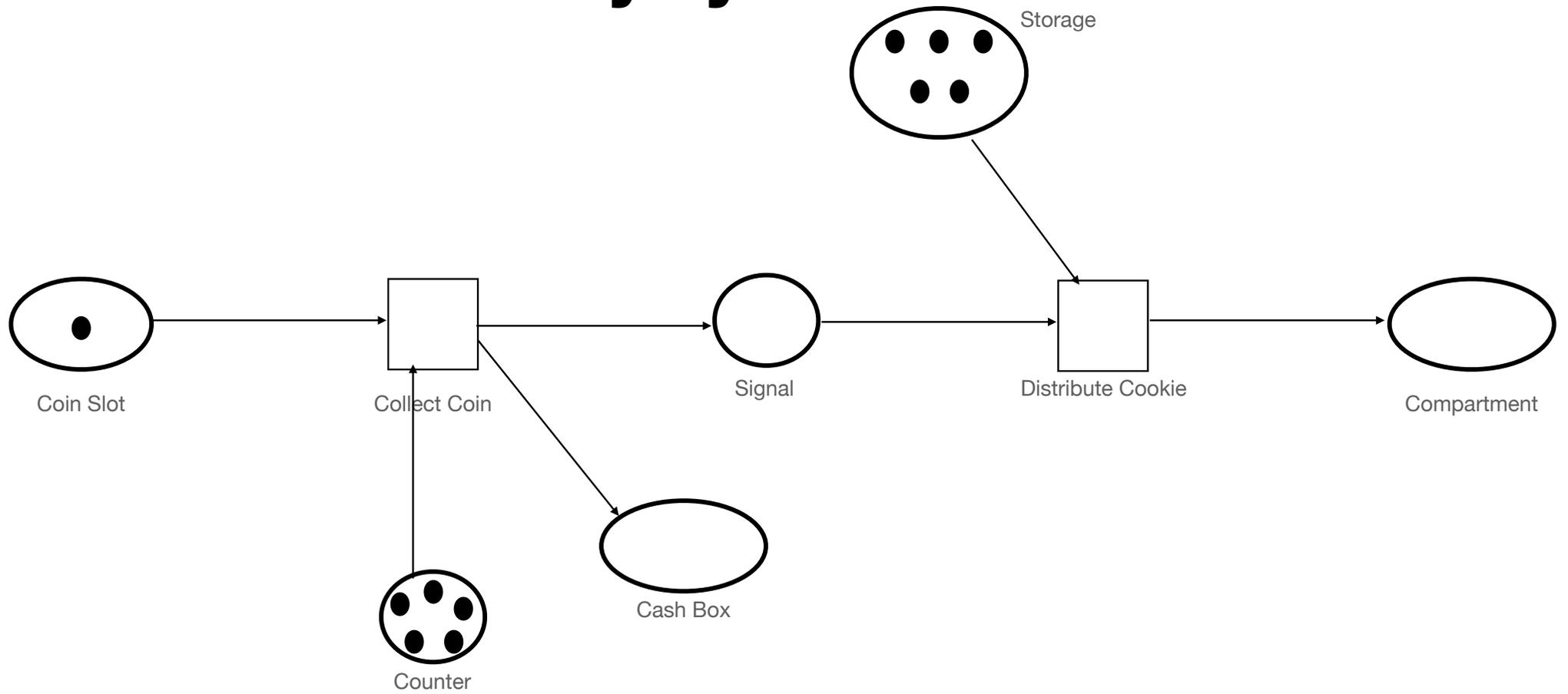
$$M_0 \xrightarrow{t_1} M_1 \xrightarrow{t_2} \dots \xrightarrow{t_n} M_n$$

(Wolfgang, 26)

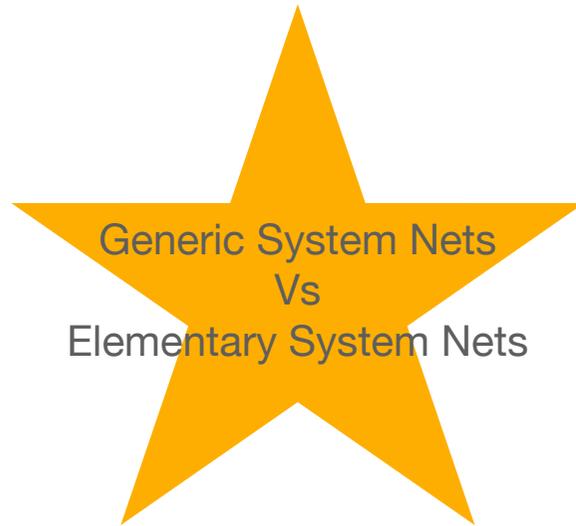
# Part 3: Elementary system nets



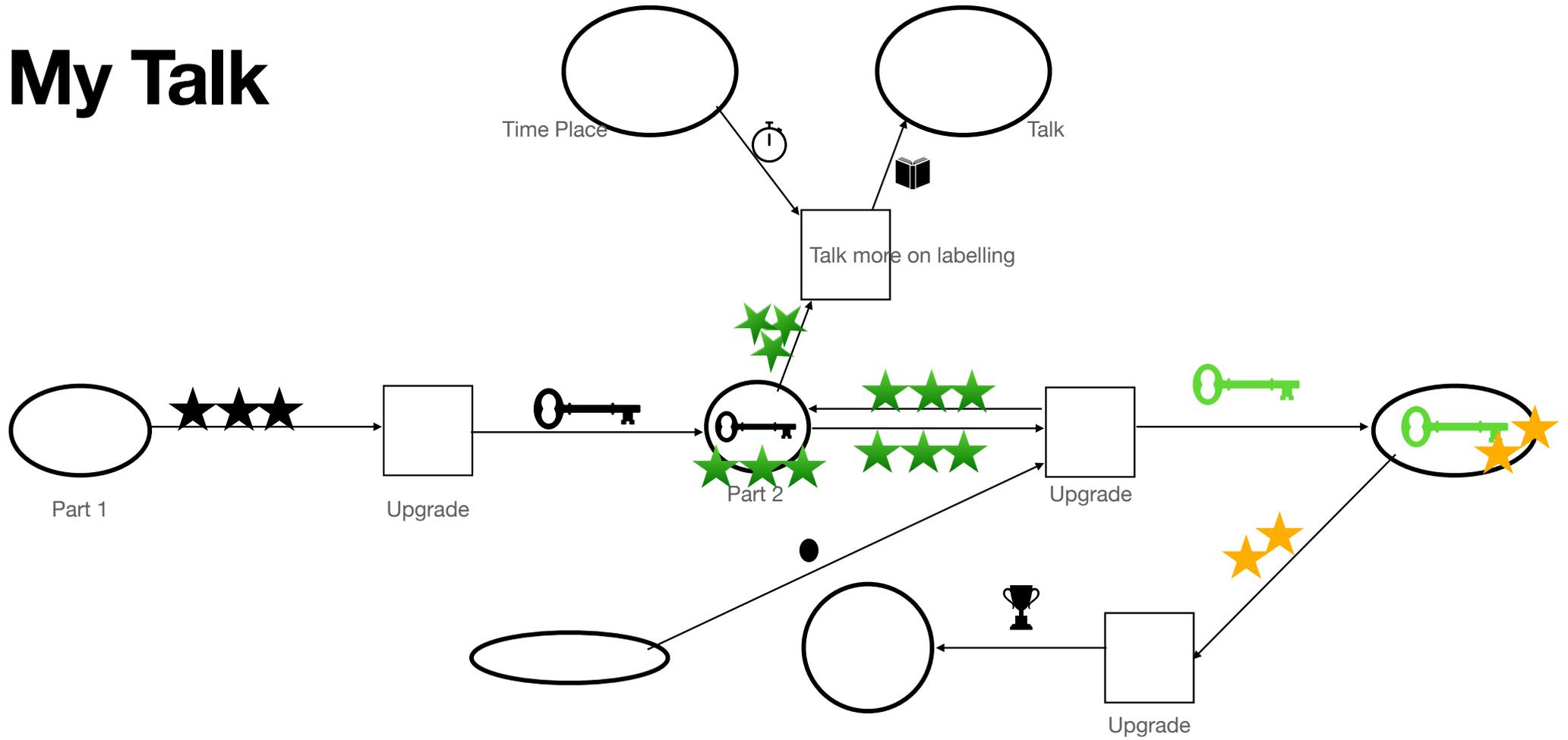
# Part 3: Elementary system nets



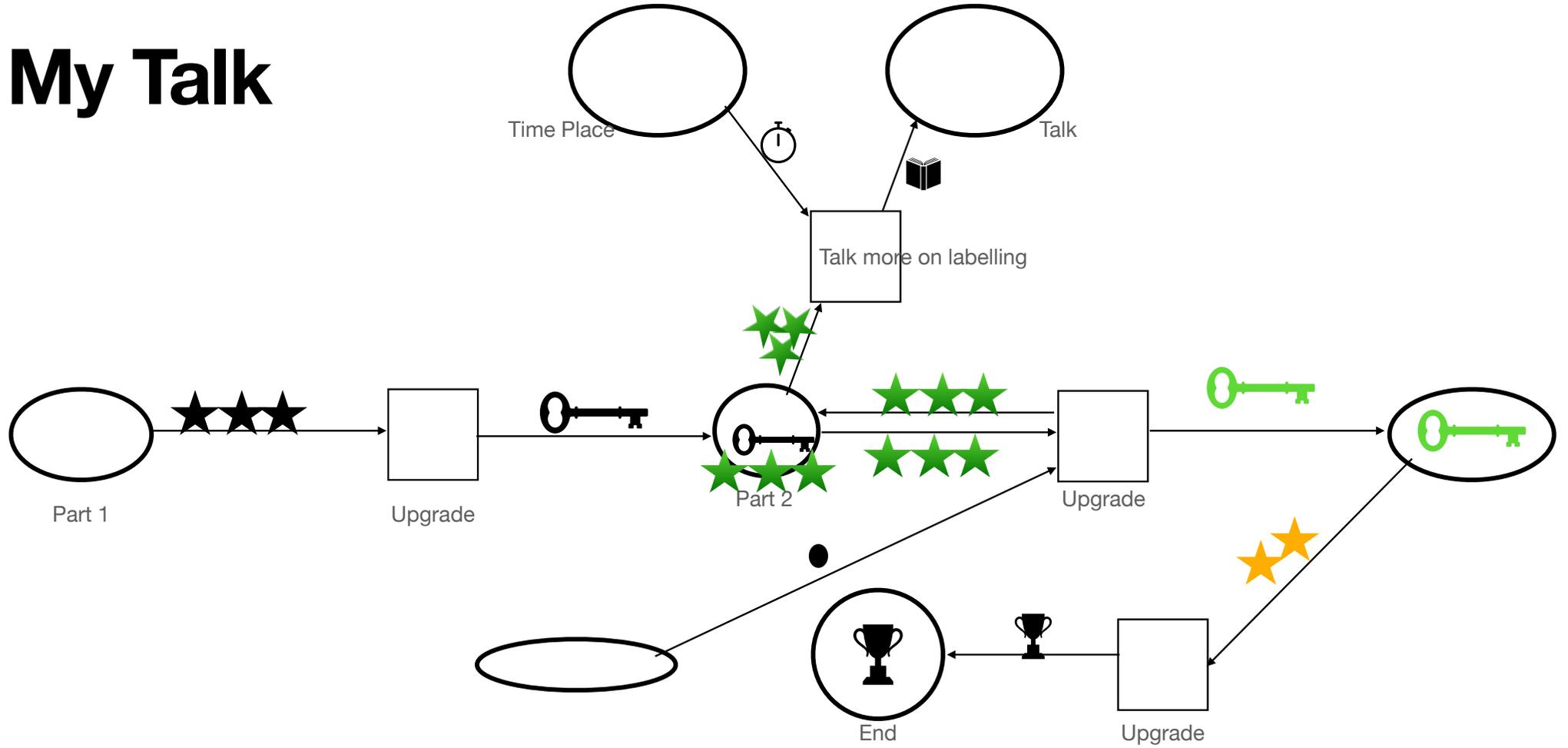
# Part 3 Take Away



# My Talk



# My Talk



**More...**

# Part 4 Sth interesting if we have time

- Labelling (Generic System Nets)
- Petri Net modelling ME
- Code and Hot Transition
- Petri Net Modelling Relay Race
- Petri Net modelling Vivian\_reading\_books\_unless\_rain
- Some interesting design in cookie vending machine

# Part 4 Sth interesting if we have time

- **Labelling of Arcs**

- “Represent the tokens that flow through the arc at the occurrence of transition.” (Wolfgang, 16)

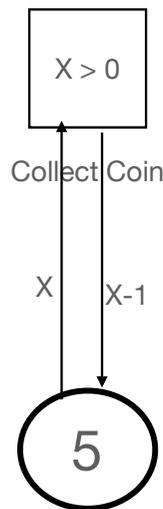
$\overline{pt}$  or  $\overline{tp}$

- NOT REAL TOKENS

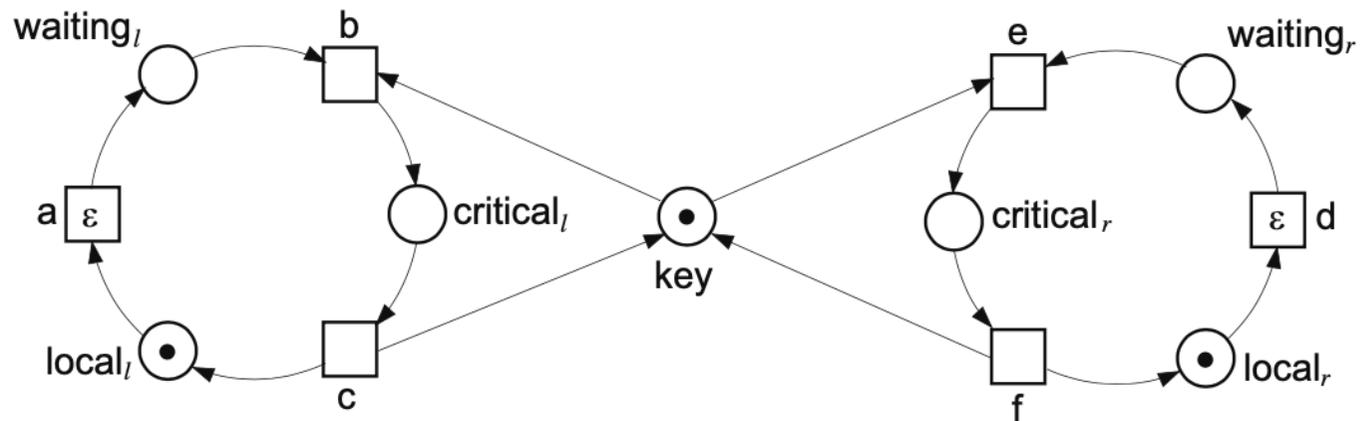
- Constant
- Variable/Function

- **Labelling of Transitions**

- Condition with variable.
- Evaluated to True or False



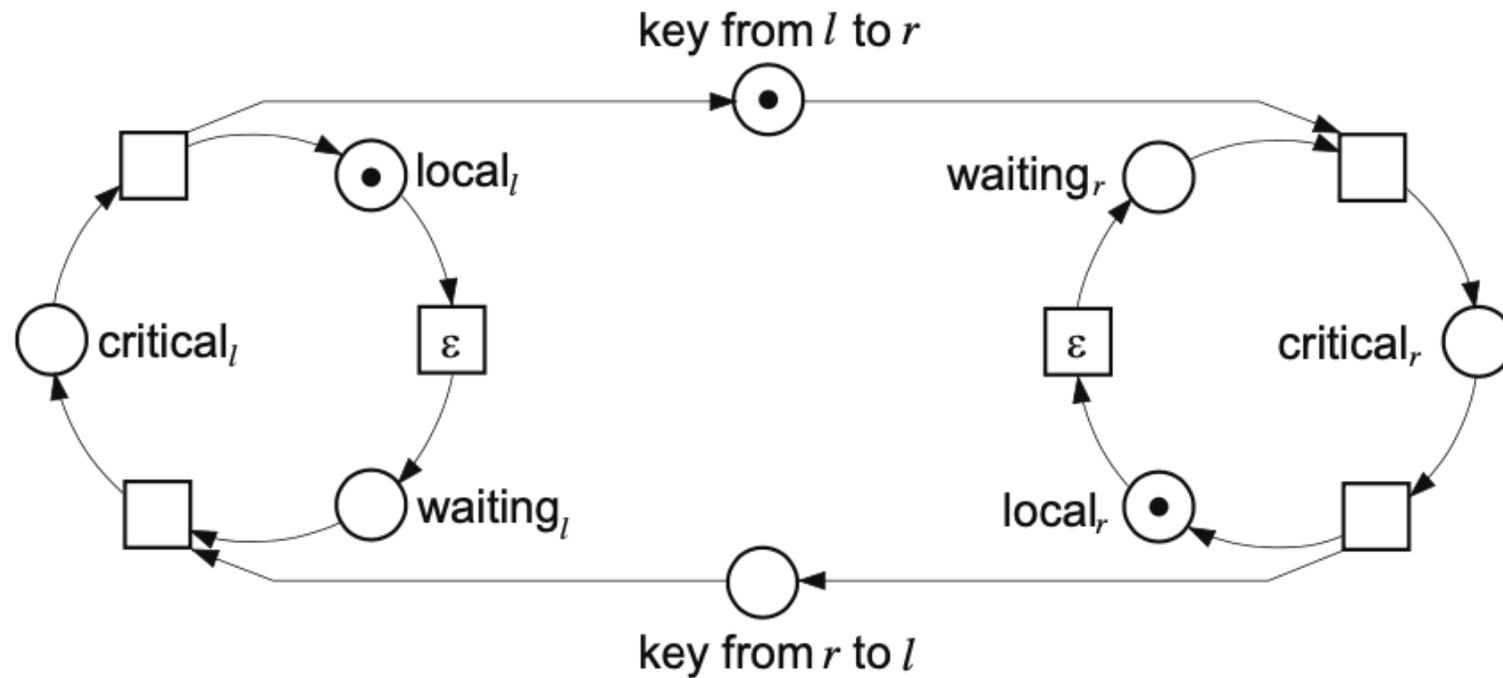
# Part 4 Sth interesting if we have time



(Wolfgang, 27)

Models ME using elementary system nets. Find sth wrong here?

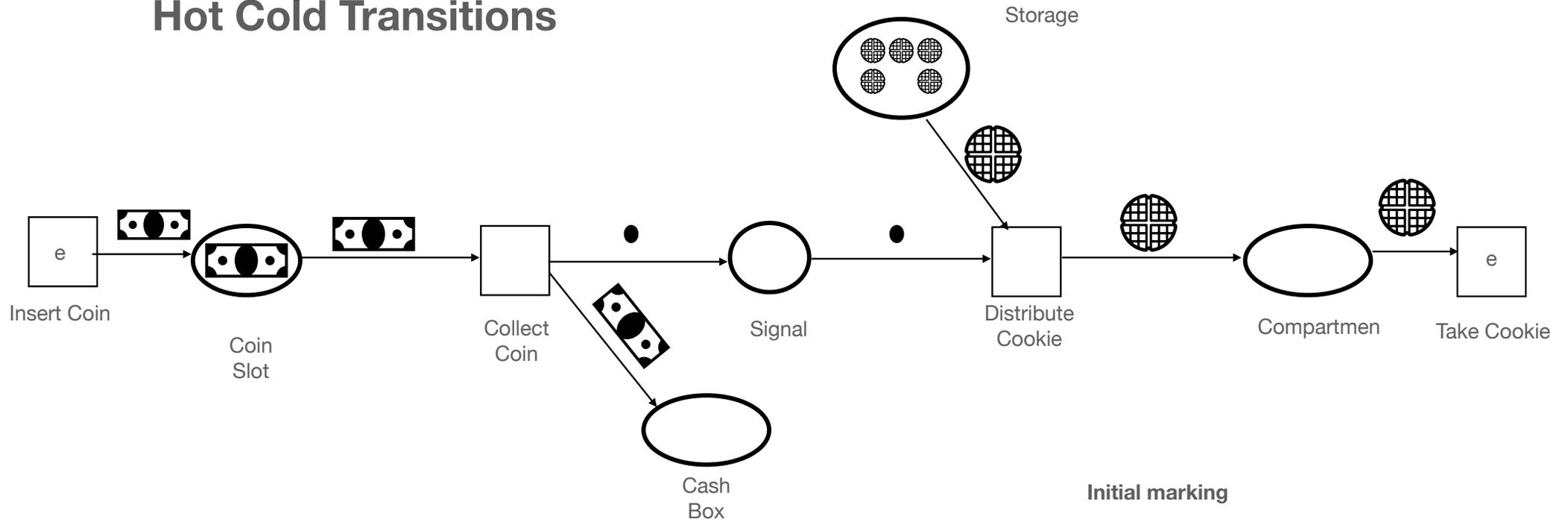
# Part 4 Sth interesting if we have time



(Wolfgang, 28)

# Part 4 Sth interesting if we have time

## Hot Cold Transitions



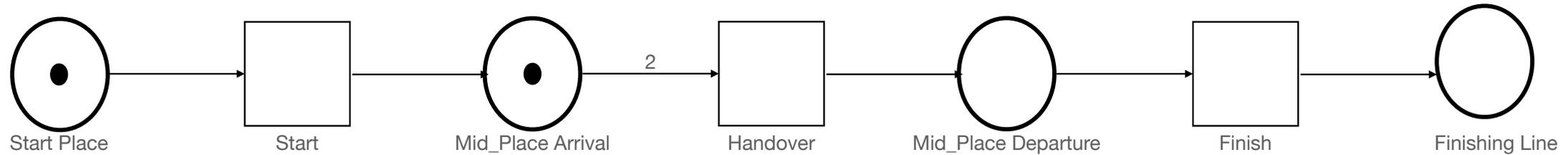
# Part 4 Sth interesting if we have time

Relay Race: Process Graph



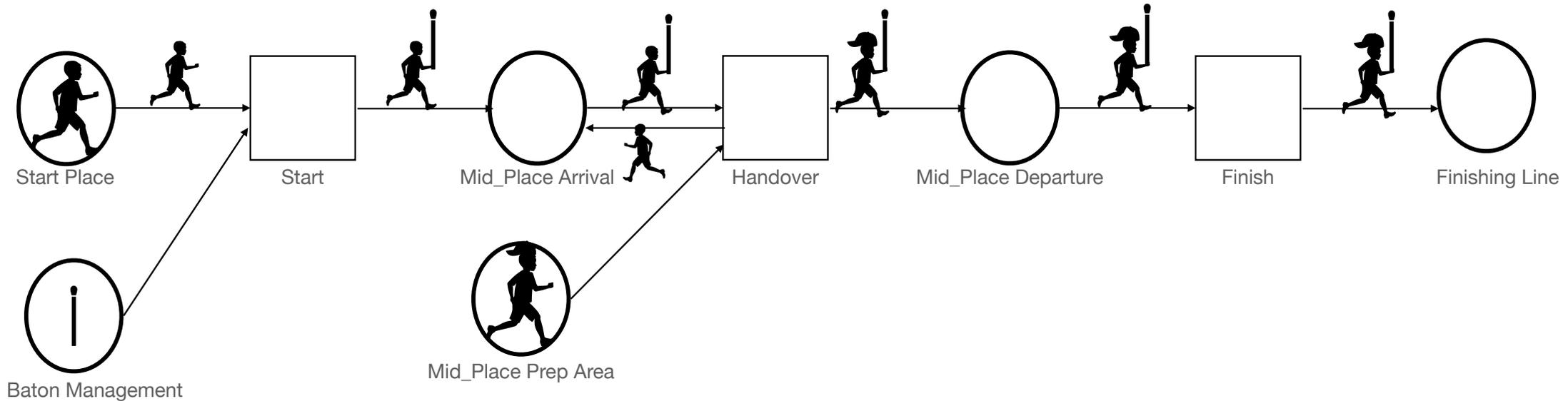
# Part 4 Sth interesting if we have time

Relay Race: Elementary System Nets

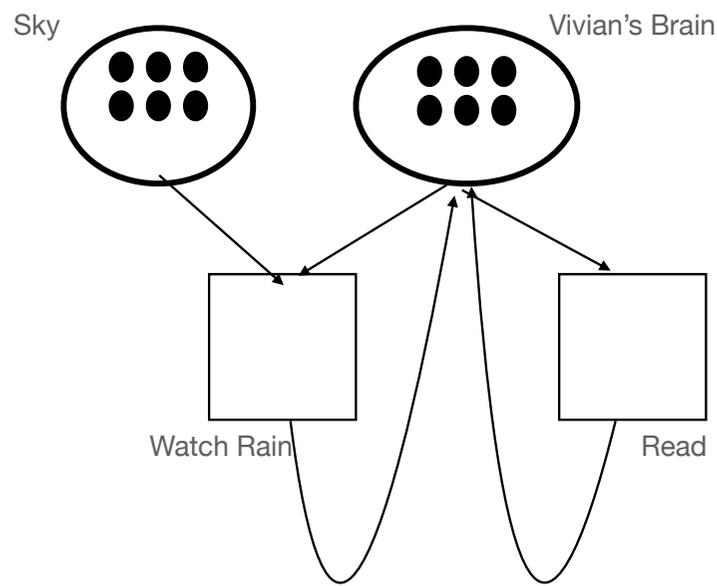


# Part 4 Sth interesting if we have time

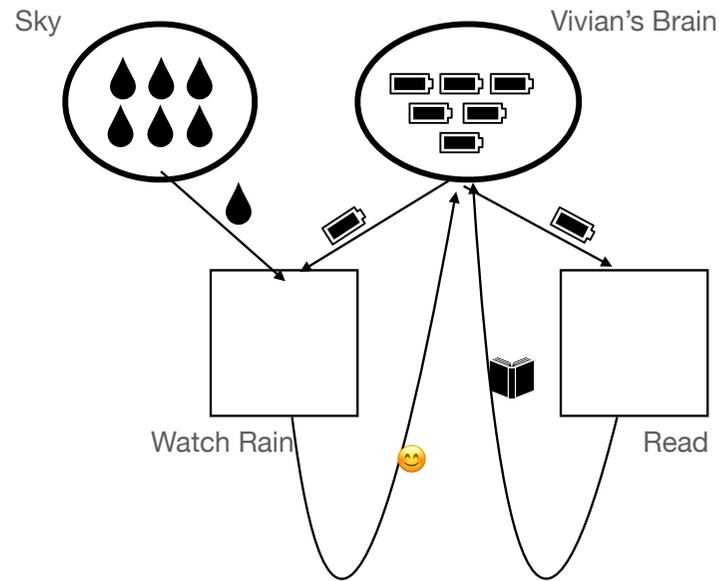
## Relay Race: Generic System Nets



# Part 4 Sth interesting if we have time



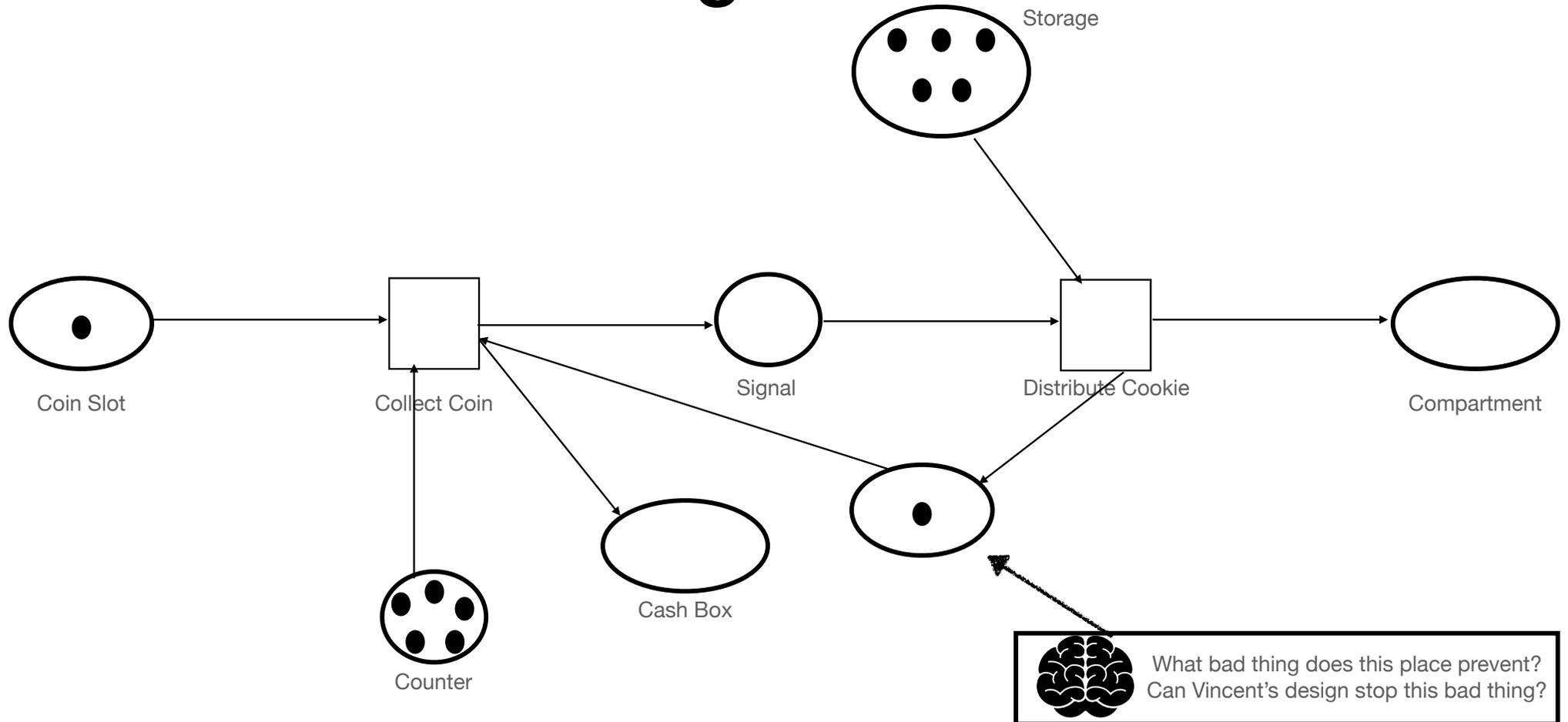
Elementary System Nets



Generic System Nets

Models Vivian's Read\_book\_unless\_rainy days

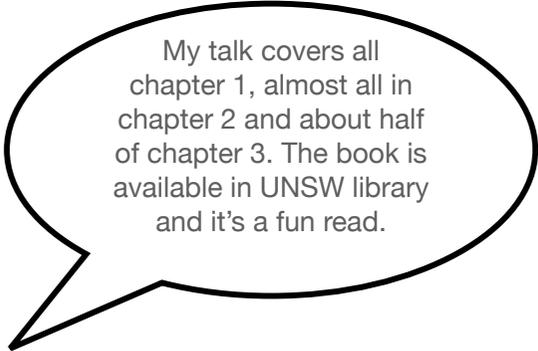
# Part 4 Sth interesting if we have time



# Links to recordings



# References



My talk covers all chapter 1, almost all in chapter 2 and about half of chapter 3. The book is available in UNSW library and it's a fun read.

Reisig, Wolfgang. Understanding Petri Nets: Modeling Techniques, Analysis Methods, Case Studies. Berlin, Heidelberg: Springer Berlin Heidelberg: Imprint: Springer, 2013. Print.

# Thank you!



Drop me an email if you got any question:  
[tiankuang.zhang@student.unsw.edu.au](mailto:tiankuang.zhang@student.unsw.edu.au)